# The t-distribution

## t test-origin:

- Founder WS Gosset
- Wrote under the pseudonym “Student”
- Mostly worked in tea (t) time
- ? Hence known as Student's *t* test.
- Certainly if $n \leq 30$

## Types:

- **One sample**
  **compare with population**
- **Unpaired**
  **compare with control**
- **Paired**
  **same subjects: pre-post**

## T-test:

1. **Test for single mean**

   Whether the sample mean is equal to the predefined population mean ?

2. **Test for difference in means**

   Whether the CD4 level of patients taking treatment A is equal to CD4 level of patients taking treatment B ?

3. **Test for paired observation**

   Whether the treatment conferred any significant benefit ?

## Test direction

- One tailed t test
- Two tailed test

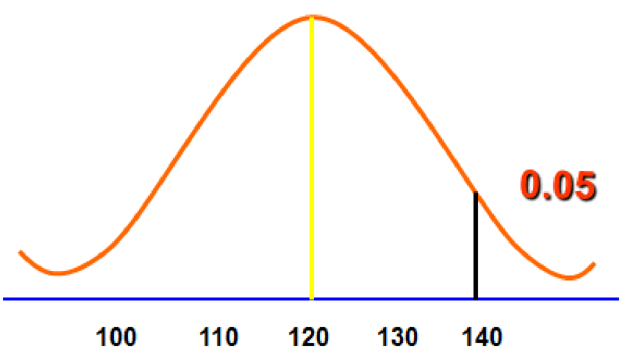
## Developing the Pooled-Variance t Test:

- **Setting Up the Hypothesis:**

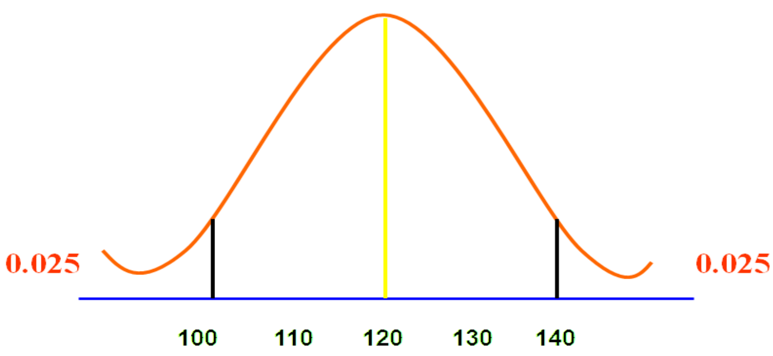
| | | | |
|---|---|---|---|
| $H_0: \mu_1 = \mu_2$<br>$H_1: \mu_1 \neq \mu_2$ | OR | $H_0: \mu_1 - \mu_2 = 0$<br>$H_1: \mu_1 - \mu_2 \neq 0$ | Two Tail |
| $H_0: \mu_1 \leq \mu_2$<br>$H_1: \mu_1 > \mu_2$ | OR | $H_0: \mu_1 - \mu_2 \leq 0$<br>$H_1: \mu_1 - \mu_2 > 0$ | Right Tail |
| $H_0: \mu_1 \geq \mu_2$<br>$H_1: \mu_1 < \mu_2$ | OR | $H_0: \mu_1 - \mu_2 \geq 0$<br>$H_1: \mu_1 - \mu_2 < 0$ | Left Tail |

Example:

**Mean systolic BP in nephritis is significantly higher than of normal person**

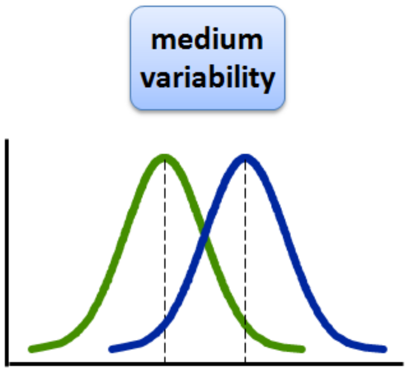


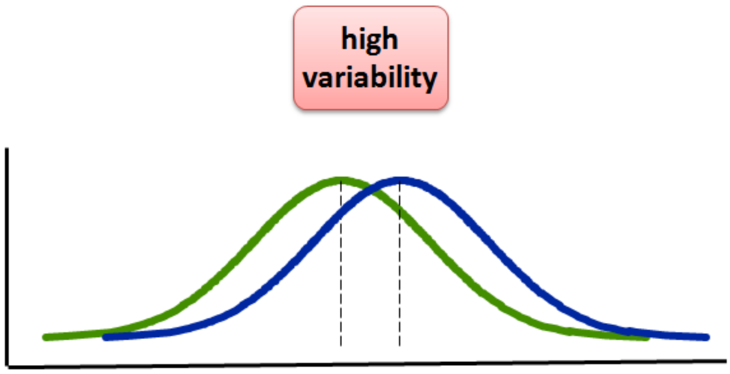

**Mean systolic BP in nephritis is significantly different from that of normal person**

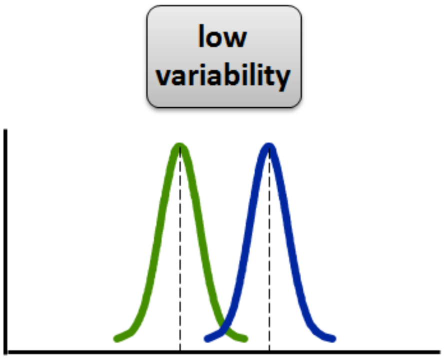


## Statistical Analysis

What does difference mean??

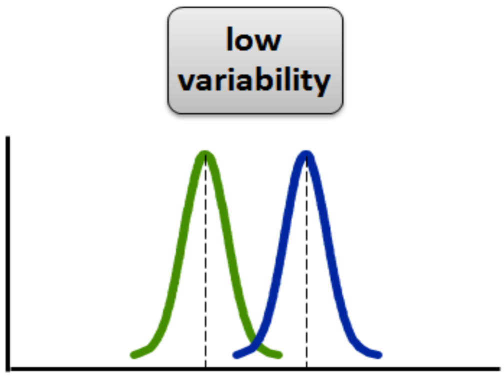



**The mean difference is the *same* for all three cases**

**Which one shows the *greatest* difference?**




- **a statistical difference is a function of the *difference between means* relative to the *variability***
- **a small difference between means with large variability could be due to *chance***
- **like a *signal-to-noise* ratio**

So we estimate

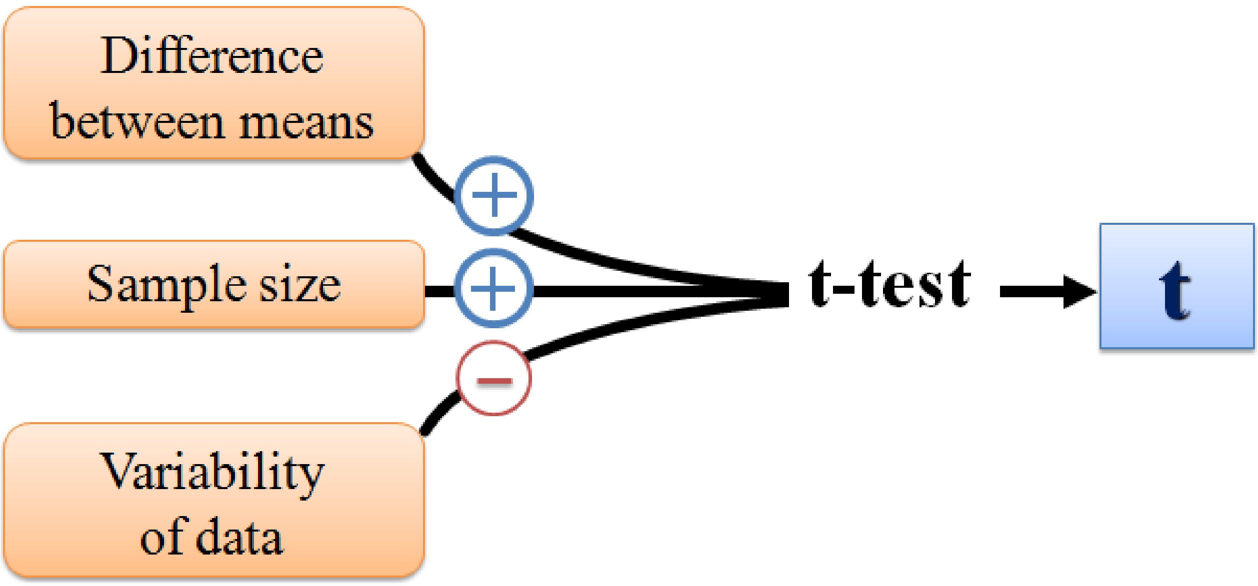
$$\frac{\text{signal}}{\text{noise}} = \frac{\text{difference between group means}}{\text{variability of groups}}$$

$$= \frac{\bar{X}_T - \bar{X}_C}{SE(\bar{X}_T - \bar{X}_C)}$$

$$= \text{t-value}$$

---

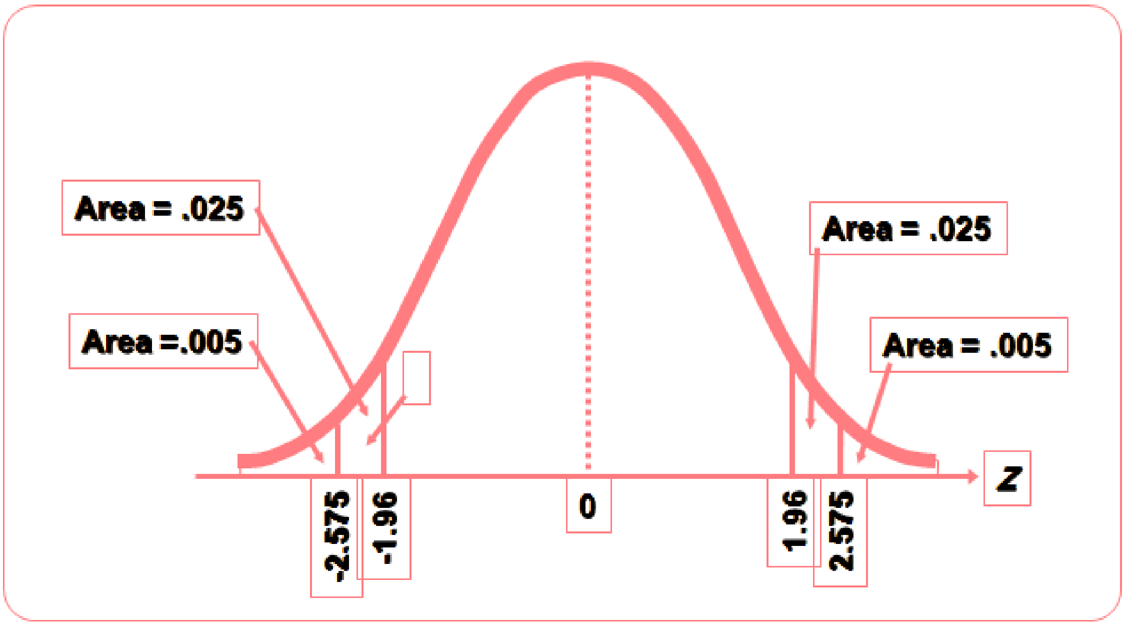
<u>**Two sample t - test**</u>

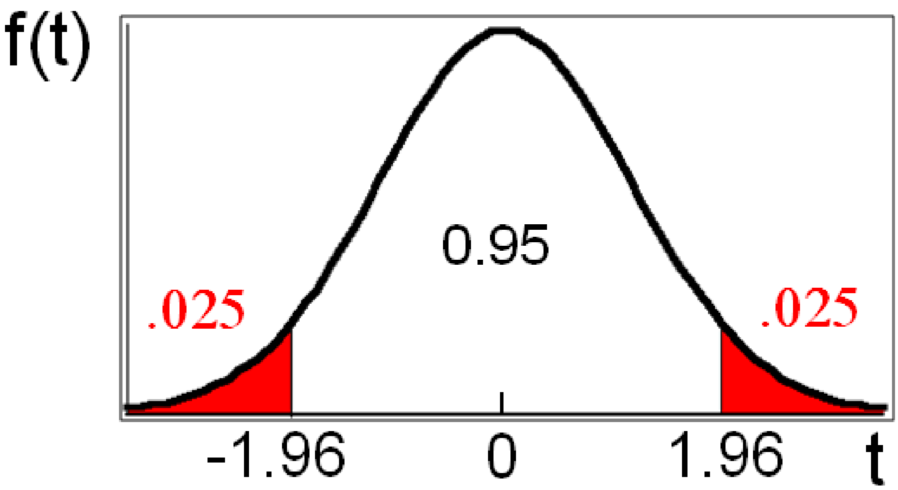


## Probability – p

- With t we check the probability
- Reject or do not reject Null hypothesis
- You reject if $p < 0.05$ or still less
- Difference between means (groups) is more & more significant if p is less & less

Determining the p-Value:




red area = rejection region for 2-sided test

**Assumptions:** when you apply t test that you have

- **Normal distribution**
- **Equal variance**
- **Random sampling**

**t-Statistic**

$$\boxed{t = \frac{\bar{x} - \mu}{s / \sqrt{n}}}$$

**When the sampled population is normally distributed, the t statistic is Student t distributed with n-1 degrees of freedom.**

## T- test for single mean:

**Example.**

**The following are the weight (mg) of each of 20 rats drawn at random from a large stock. Is it likely that the mean weight of these 20 rats are similar to the mean weight ( 24 mg ) of the whole stock ?**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9 | 18 | 21 | 26 | 14 | 18 | 22 | 27 |
| 15 | 19 | 22 | 29 | 15 | 19 | 24 | 30 |
| | | 16 | 20 | 24 | 32 | | |

**Steps for test for single mean:**

1. **Questioned to be answered**

   Is the Mean weight of the sample of 20 rats is 24 mg?

   N=20, $\overline{X}$ =21.0 mg , S.D = 5.91 , μ =24.0 mg

2. **Null Hypothesis**

   The mean weight of rats is 24 mg. That is, The sample mean is equal to population mean.

3. **Test statistics** $t = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$ **--- $t_{(n-1)}$ df**

4. **Comparison with theoretical value**

   if tab $t_{(n-1)}$ < cal $t_{(n-1)}$ reject Ho,

   if tab $t_{(n-1)}$ > cal $t_{(n-1)}$ accept Ho,

5. **Inference**

So.

**Test statistics :** N=20, $\overline{X}$ =21.0 mg , S.D = 5.91 , μ =24.0 mg

$$t = \frac{l21.0 - 24l}{5.91 / \sqrt{20}} = -2.30$$

$t_\alpha = t_{.05, 19} = 2.093$ Accept $H_0$ if t < 2.093

Reject $H_0$ if t >= 2.093

**Inference :**

**There is no evidence that the sample is taken from the population with mean weight of 24 gm.**

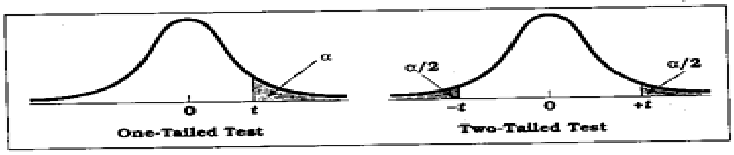



**Table D.6** **Percentage Points of the *t* Distribution** (Source: The entries in this table were computed by the author.)

| | Level of Significance for One-Tailed Test | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | .25 | .20 | .15 | .10 | .05 | .025 | .01 | .005 | .0005 |
| | Level of Significance for Two-Tailed Test | | | | | | | | |
| *df* | .50 | .40 | .30 | .20 | .10 | .05 | .02 | .01 | .001 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 63.662 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 40 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 50 | 0.679 | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.496 |
| 100 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.390 |
| ∞ | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 |

---

## T-test for difference in means:

Example.

Given below are the 24 hrs total energy expenditure (MJ/day) in groups of lean and obese women. Examine whether the obese women's mean energy expenditure is significantly higher ?

**Lean**

| | | |
|---|---|---|
| 6.1 | 7.0 | 7.5 |
| 7.5 | 5.5 | 7.6 |
| 7.9 | 8.1 | 8.1 |
| 8.1 | 8.4 | 10.2 |
| | 10.9 | |

**Obese**

| | | |
|---|---|---|
| 8.8 | 9.2 | 9.2 |
| 9.7 | 9.7 | 10.0 |
| 11.5 | 11.8 | 12.8 |

- **Null Hypothesis:**

Obese women's mean energy expenditure is qual to the lean women's energy expenditure.

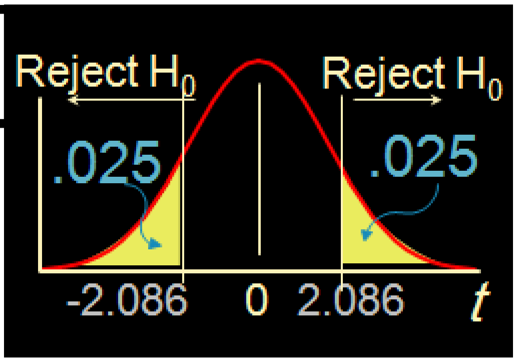
Data summary

| | Lean | Obeses |
|---|---|---|
| N | 13 | 9 |
| | 8.10 | 10.30 |
| S | 1.38 | 1.25 |

- **Solution:**

- $H_0$: $\mu_1 - \mu_2 = 0$ $(\mu_1 = \mu_2)$
- $H_1$: $\mu_1 - \mu_2 \neq 0$ $(\mu_1 \neq \mu_2)$
- $\alpha = 0.05$
- df = 13 + 9 - 2 = 20

**Critical Value(s):**




- **Compute the Test Statistic:**

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \bullet \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Hypothesized Difference (usually zero when testing for equal means)

$$df = n_1 + n_2 - 2$$

- **Calculate the Pooled Sample Variances as an Estimate of the Common Populations Variance:**

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$S_p^2$ = Pooled-Variance

$S_1^2$ = Variance of Sample 1

$S_2^2$ = Variance of sample 2

$n_1$ = Size of Sample 1

$n_2$ = Size of Sample 2

First, estimate the common variance as a weighted average of the two sample variances using the degrees of freedom as weights

$$S_p^{\,2} = \frac{(13-1)\,.\;1.38^2 + (9-1)\,.\;1.25^2}{(13-1)-(9-1)} = 1.765$$

- Calculating the test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \,.\, \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(8.1 - 10.3) - 0}{\sqrt{1.76 \,.\, \left(\frac{1}{13} + \frac{1}{9}\right)}} = 3.82$$

**tab t 9+13-2 =20 dff = t 0.05,20 =2.086**

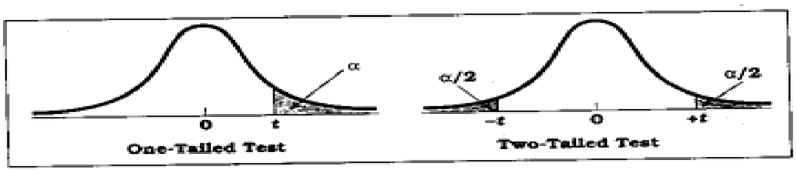



**Table D.6** **Percentage Points of the *t* Distribution** (Source: The entries in this table were computed by the author.)

| | Level of Significance for One-Tailed Test | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | .25 | .20 | .15 | .10 | .05 | .025 | .01 | .005 | .0005 |
| | Level of Significance for Two-Tailed Test | | | | | | | | |
| *df* | .50 | .40 | .30 | .20 | .10 | .05 | .02 | .01 | .001 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 63.662 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 40 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 50 | 0.679 | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.496 |
| 100 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.390 |
| ∞ | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 |

## T-test for difference in means:

Inference : The cal t (3.82) is higher than tab t at 0.05, 20. i.e. 2.086 . This implies that there is a evidence that the mean energy expenditure in obese group is significantly ($p<0.05$) higher than that of lean group.

Example.

Suppose we want to test the effectiveness of a program designed to increase scores on the quantitative section of the Graduate Record Exam (GRE). We test the program on a group of 8 students. Prior to entering the program, each student takes a practice quantitative GRE; after completing the program, each student takes another practice exam. Based on their performance, was the program effective?

- **Each subject contributes 2 scores: repeated measures design**

| Student | Before Program | After Program |
|---|---|---|
| 1 | 520 | 555 |
| 2 | 490 | 510 |
| 3 | 600 | 585 |
| 4 | 620 | 645 |
| 5 | 580 | 630 |
| 6 | 560 | 550 |
| 7 | 610 | 645 |
| 8 | 480 | 520 |

- **Can represent each student with a single score: the difference (D) between the scores:**

| Student | Before Program | After Program | D |
|---|---|---|---|
| 1 | 520 | 555 | 35 |
| 2 | 490 | 510 | 20 |
| 3 | 600 | 585 | -15 |
| 4 | 620 | 645 | 25 |
| 5 | 580 | 630 | 50 |
| 6 | 560 | 550 | -10 |
| 7 | 610 | 645 | 35 |
| 8 | 480 | 520 | 40 |

- **ApproacH:**

  test the effectiveness of program by testing significance of D

- **Null hypothesis:**

  There is no difference in the scores of before and after program

- **Alternative hypothesis:**

  program is effective → scores after program will be higher than scores before program → average D will be greater than zero

$$H_0: \mu_D = 0$$

$$H_1: \mu_D > 0$$

| Student | Before Program | After Program | D | $D^2$ |
|---|---|---|---|---|
| 1 | 520 | 555 | 35 | 1225 |
| 2 | 490 | 510 | 20 | 400 |
| 3 | 600 | 585 | -15 | 225 |
| 4 | 620 | 645 | 25 | 625 |
| 5 | 580 | 630 | 50 | 2500 |
| 6 | 560 | 550 | -10 | 100 |
| 7 | 610 | 645 | 35 | 1225 |
| 8 | 480 | 520 | 40 | 1600 |
| | | | $\sum D = 180$ | $\sum D^2 = 7900$ |

**Recall that**

| **For single samples:** | **For related samples:** |
|---|---|
| $t_{obt} = \dfrac{\overline{X} - \mu}{s_{\overline{X}}} = \dfrac{\text{score - mean}}{\text{standard error}}$ | $t_{obt} = \dfrac{\overline{D} - \mu_D}{s_{\overline{D}}}$ |

<u>where:</u>

$$s_{\overline{D}} = \frac{s_D}{\sqrt{N}} \quad \text{and} \quad s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{N}}{N-1}}$$

**Mean of D:**

$$\overline{D} = \frac{\sum D}{N} = \frac{180}{8} = 22.5$$

**Standard deviation of D:**

$$s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{N}}{N-1}} = \sqrt{\frac{7900 - \frac{(180)^2}{8}}{8-1}} = 23.45$$

**Standard error:**

$$s_{\overline{D}} = \frac{s_D}{\sqrt{N}} = \frac{23.45}{\sqrt{8}} = 8.2908$$

---

$$t_{obt} = \frac{\overline{D} - \mu_D}{s_{\overline{D}}}$$

Under $H_0$, $\mu_D = 0$, so:

$$t_{obt} = \frac{\overline{D}}{s_{\overline{D}}} = \frac{22.5}{8.2908} = 2.714$$

From Table ( next page ): for $\alpha = 0.05$, one-tailed, with df = 7

$$t_{critical} = 1.895$$

$2.714 > 1.895 \rightarrow$ reject $H_0$

**The program is effective**

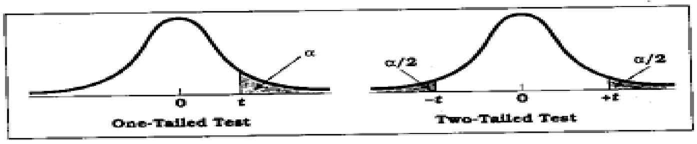



**Table D.6** **Percentage Points of the *t* Distribution** (Source: The entries in this table were computed by the author.)

| | Level of Significance for One-Tailed Test | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | .25 | .20 | .15 | .10 | .05 | .025 | .01 | .005 | .0005 |
| | Level of Significance for Two-Tailed Test | | | | | | | | |
| *df* | .50 | .40 | .30 | .20 | .10 | .05 | .02 | .01 | .001 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 63.662 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 40 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 50 | 0.679 | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.496 |
| 100 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.390 |
| ∞ | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.291 |

## t-Value:

t is a measure of: How difficult is it to believe the null hypothesis?

High t

Difficult to believe the null hypothesis - accept that there is a real difference.

Low t

Easy to believe the null hypothesis - have not proved any difference.

In Conclusion

**Student ‘s t-test will be used:**

- **When Sample size is small**
- **and for the following situations:**
  - to compare the single sample mean with the population mean
  - to compare the sample means of two indpendent samples
  - to compare the sample means of paired samples