Statistical tests for Quantitative variables (z-test, t-test & Correlation)

BY

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Objectives:

- (1) Able to understand the factors to apply for the choice of statistical tests in analyzing the data .
- (2) Able to apply appropriately Z-test, student’s t-test & Correlation
- (3) Able to interpret the findings of the analysis using these three tests.

Choosing the appropriate Statistical test

- Based on the three aspects of the data
  - Type of variables
  - Number of groups being compared &
  - Sample size

Statistical Tests

Z-test:

Study variable: Qualitative (Categorical)

Outcome variable: Quantitative

Comparison:

(i)sample mean with population mean

(ii)two sample means

Sample size: larger in each group(>30) & standard deviation is known

Steps for Hypothesis Testing

Formulate H_0 and H_1

Select Appropriate Test

Choose Level of Significance

Calculate Test Statistic TS_{CAL}

(1)Determine Critical Value TS_{CR}

(2) Determine if TS_{CL} falls into (Non) Rejection Region

(3) Determine Prob Assoc with Test Stat

(4) Compare with Level of Significance, α

Reject/Do not Reject H_0

Draw Research Conclusion

Example(Comparing Sample mean with Population mean):

- The education department at a university has been accused of “grade inflation” in medical students with higher GPAs than students in general.

- GPAs of all medical students should be compared with the GPAs of all other (non-medical) students.
  - There are 1000s of medical students, far too many to interview.
  - How can this be investigated without interviewing all medical students ?

What we know:

- The average GPA for ***all* other** students is 2.70. This value is a **parameter.**

$\mu = 2.70$

- To the right is the statistical information for a random sample of medical students:

$\bar{X}$ =	3.00
s =	0.70
n =	117

Questions to ask:

- Is there a difference between the parameter (2.70) and the statistic (3.00)?
- Could the observed difference have been caused by random chance?
- Is the difference real (significant)?

1. The sample mean (3.00) is the same as the pop. mean (2.70).
   - The difference is trivial and caused by random chance.

2. The difference is real (significant).
   - Medical students are different from all other students.

Step 1: Make Assumptions and Meet Test Requirements

- Random sampling
  - Hypothesis testing assumes samples were selected using random sampling.
  - In this case, the sample of 117 cases was randomly selected from all medical students.

- Level of Measurement is Ratio scale
  - GPA is measured on ration scale, so the mean is an appropriate statistic.
- Sampling Distribution is normal in shape
  - This is a “large” sample ($n \geq 100$).

Step 2 State the Null Hypothesis

- H_0: μ= $\bar{X}$
  - We can state H_o: No difference between the sample mean and the population parameter
  - (In other words, the sample mean of 3.0 really the same as the population mean of 2.7 – the difference is not real but is due to chance.)
  - The sample of 117 comes from a population that has a GPA of 2.7.
  - The difference between 2.7 and 3.0 is trivial and caused by random chance.

Step 2 (cont.) State the Alternative Hypothesis

- H_1: $\mu \neq \bar{X}$
  - Or H_1: There is a difference between the sample mean and the population parameter
  - The sample of 117 comes a population that *does not* have a GPA of 2.7. In reality, it comes from a different population.
  - The difference between 2.7 and 3.0 reflects an actual difference between medical students and other students.
  - Note that we are testing whether the population the sample comes from is from a different population or is the same as the general student population.

Step 3 Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution= Z
  - Alpha (α) = .05
  - α is the indicator of "rare" events.
  - Any difference with a probability less than α is rare and will cause us to reject the H_0.

Step 3 (cont.) Select Sampling Distribution and Establish the Critical Region

- Critical Region begins at $Z = \pm 1.96$
  - This is the critical Z score associated with $\alpha = .05$, two-tailed test.
  - If the obtained Z score falls in the Critical Region, or "the region of rejection," then we would reject the H_0.

When the Population σ is not known, use the following formula:

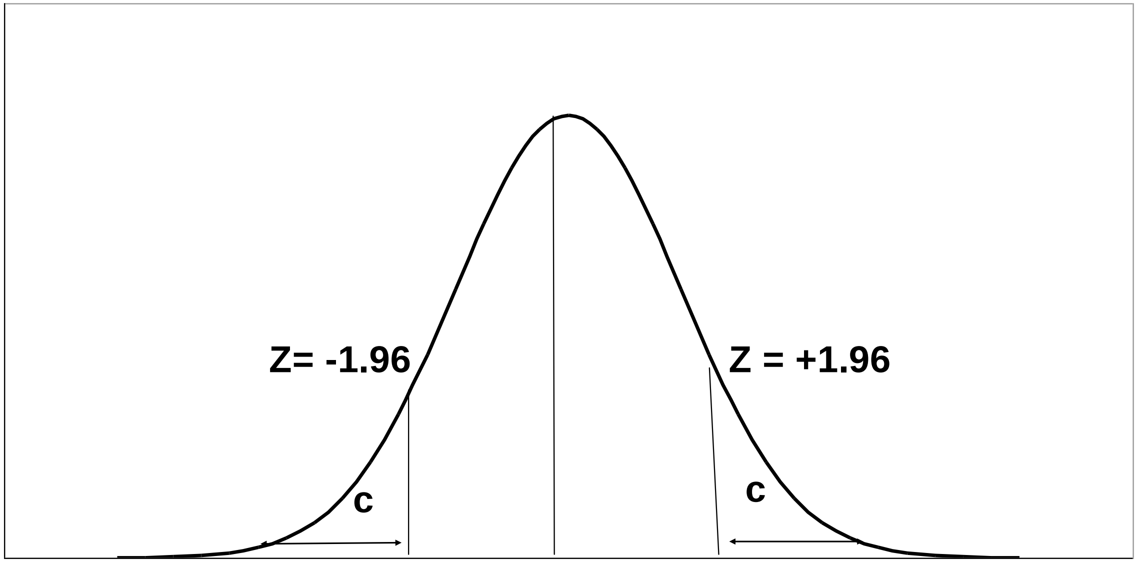
$$Z = \frac{\bar{X} - \mu}{s / \sqrt{n-1}}$$

Test the Hypotheses

$$Z = \frac{3.0 - 2.7}{.7 \big/ \sqrt{117 - 1}} = 4.62$$

- Substituting the values into the formula, we calculate a Z score of 4.62.

Two-tailed Hypothesis Test



When α = .05, then .025 of the area is distributed on either side of the curve in area **(C)**

The .95 in the **middle section** represents **no significant difference** between the population and the sample mean.

The cut-off between the middle section and +/- .025 is represented by a **Z-value of +/- 1.96.**

Step 5 Make a Decision and Interpret Results

- The obtained Z score fell in the Critical Region, so we *reject* the H_0.
  - If the H_0 were true, a sample outcome of 3.00 would be unlikely.
  - Therefore, the H_0 is false and must be rejected.

- Medical students have a GPA that is significantly different from the non-medical students ($Z = 4.62$, $p < 0.05$).

Summary:

- The GPA of medical students is *significantly* different from the GPA of non-medical students.

- In hypothesis testing, we try to identify statistically significant differences that did not occur by random chance.

- In this example, the difference between the parameter 2.70 and the statistic 3.00 was large and unlikely ($p < .05$) to have occurred by random chance.

Comparison of two sample means

Example : Weight Loss for Diet vs Exercise

Did dieters lose more fat than the exercisers?

Diet Only:

sample mean = 5.9 kg

sample standard deviation = 4.1 kg

sample size = *n* = 42

standard error = $SEM_1 = 4.1/\sqrt{42} = 0.633$

Exercise Only:

sample mean = 4.1 kg

sample standard deviation = 3.7 kg

sample size = *n* = 47

standard error = $SEM_2 = 3.7/\sqrt{47} = 0.540$

measure of variability = $\sqrt{[(0.633)^2 + (0.540)^2]} = 0.83$

Example : Weight Loss for Diet vs Exercise

Step 1. Determine the null and alternative hypotheses.

Null hypothesis: No difference in average fat lost for two methods. Sample mean difference is ***zero***.

Alternative hypothesis*:** There is a difference in average fat lost in sample for two methods. Sample mean difference is not ***zero.

Step 2. Sampling distribution: Normal distribution (z-test)

Step 3. Assumptions of test statistic (sample size > 30 in each group)

Step 4. Collect and summarize data into a test statistic.

The sample mean difference = 5.9 – 4.1 = 1.8 kg
and the standard error of the difference is 0.83.

So the *test statistic:* $z = \dfrac{1.8 - 0}{0.83} = 2.17$

Example : Weight Loss for Diet vs Exercise

Step 5. Determine the *p*-value.

Recall the alternative hypothesis was two-sided.

p-value = 2 × [proportion of bell-shaped curve above 2.17]

Z-test table => proportion is about 2 × 0.015 = 0.03.

Step 6. Make a decision.

The ***p*-value of 0.03 is less than or equal to 0.05**, so …

- If really no difference between dieting and exercise as fat loss methods, would see such an extreme result only 3% of the time, or 3 times out of 100.
- Prefer to believe truth does not lie with null hypothesis. We conclude that there is a ***statistically significant difference between average fat loss for the two methods***.

Student's t-test:

Study variable: Qualitative (Categorical)

Outcome variable: Quantitative

Comparison:

(i)sample mean with population mean
(ii)two means (independent samples)
(iii)paired samples

Sample size: each group <30 (can be used even for large sample size)

Student's t-test

1. **Test for single mean** (Student's t-test for single mean)
   Whether the sample mean is equal to the predefined population mean ?

2. **Test for difference in means (Student's t-test for independent samples)**
   Whether the CD4 level of patients taking treatment A is equal to CD4 level of patients taking treatment B ?

3. **Test for paired observation (Student's t-test for dependent samples)**
   Whether the treatment conferred any significant benefit ?

Steps for test for single mean

1. Questioned to be answered

   Is the Mean weight of the sample of 20 rats is 24 mg?

   N=20, $\bar{x}$=21.0 mg, sd=5.91 , μ =24.0 mg

2. Null Hypothesis

   The mean weight of rats is 24 mg. That is, The sample mean is equal to population mean.

3. Test statistics $$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$ --- $t_{(n-1)}$ df

4. Comparison with theoretical value

   if tab $t_{(n-1)}$ < cal $t_{(n-1)}$ reject Ho,

   if tab $t_{(n-1)}$ > cal $t_{(n-1)}$ accept Ho,

5. Inference

t –test for single mean

- **Test statistics**

n=20, $\bar{x}$ =21.0 mg, sd=5.91 ,

μ =24.0 mg

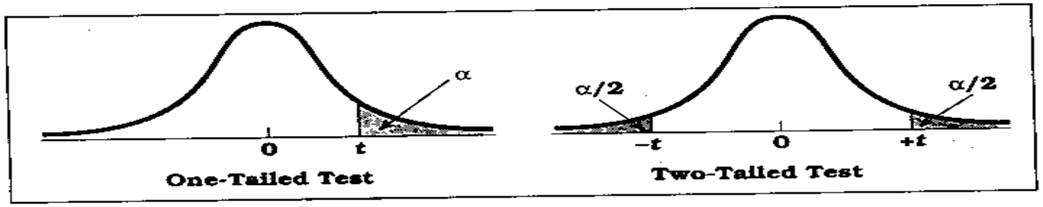
$$t = \frac{|21.0 - 24|}{5.91/\sqrt{20}} = 2.30$$

$t_{\alpha} = t_{.05,\ 19} = 2.093$

Accept H_0 if $t < 2.093$

Reject H_0 if $t >= 2.093$

Inference : We reject Ho, and conclude that the data is not providing enough evidence, that the sample is taken from the population with mean weight of 24 gm



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	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

t-test for difference in means

Given below are the 24 hrs total energy expenditure (MJ/day) in groups of lean and obese women. Examine whether the obese women's mean energy expenditure is significantly higher ?.

Lean		
6.1	7.0	7.5
7.5	5.5	7.6
7.9	8.1	8.1
8.1	8.4	10.2
10.9		

Obese		
8.8	9.2	9.2
9.7	9.7	10.0
11.5	11.8	12.8

Null Hypothesis

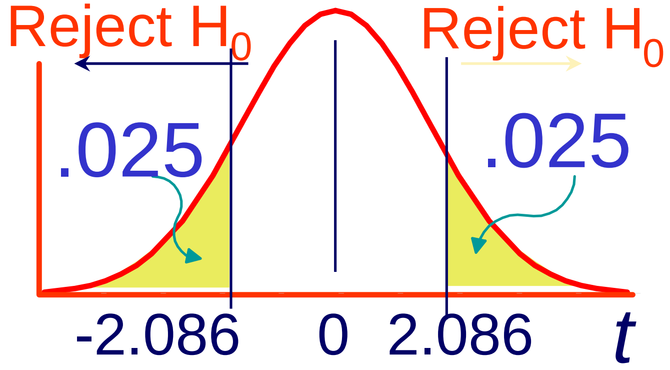
Obese women's mean energy expenditure is equal to the lean women's energy expenditure.

Data Summary

	lean	**Obese**
N	**13**	**9**
	8.10	**10.30**
S	**1.38**	**1.25**

Solution

- H_0: $\mu_1 - \mu_2 = 0$ ($\mu_1 = \mu_2$)
- H_1: $\mu_1 - \mu_2 \neq 0$ ($\mu_1 \neq \mu_2$)
- $\alpha = 0.05$
- df = 13 + 9 - 2 = 20
- Critical Value(s):



Calculating the Test Statistic:

- **Compute the Test Statistic:**

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2 \bullet \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Hypothesized Difference (usually zero when testing for equal means)

$$df = n_1 + n_2 - 2$$

$$S_P^2 = \frac{\left(n_1 - 1\right) \times S_1^2 + \left(n_2 - 1\right) \times S_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}$$

Developing the Pooled-Variance t Test

•Calculate the Pooled Sample Variances as an Estimate of the Common Populations Variance:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

S_p^2 = Pooled-Variance

S_1^2 = Variance of Sample 1

S_2^2 = Variance of sample 2

n_1 = Size of Sample 1

n_2 = Size of Sample 2

First, estimate the common variance as a weighted average of the two sample variances using the degrees of freedom as weights

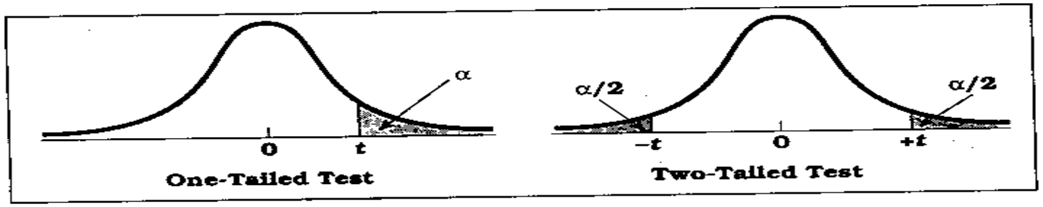
$$S_P^2 = \frac{(n_1 - 1) \times S_1^2 + (n_2 - 1) \times S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$= \frac{(13 - 1) \times 1.38^2 + (9 - 1) \times 1.25^2}{(13 - 1) + (9 - 1)} = 1.765$$

Calculating the Test Statistic:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^2 \bullet \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{|8.1 - 10.3| - 0}{176\sqrt{\frac{1}{13} + \frac{1}{9}}} = 3.82$$

tab t 9+13-2 =20 dff = t 0.05,20 =2.086



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∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

T-test for difference in means

Inference : The cal t (3.82) is higher than tab t at 0.05, 20. ie 2.086 . This implies that there is a evidence that the mean energy expenditure in obese group is significantly ($p<0.05$) higher than that of lean group

Example

Suppose we want to test the effectiveness of a program designed to increase scores on the quantitative section of the Graduate Record Exam (GRE). We test the program on a group of 8 students. Prior to entering the program, each student takes a practice quantitative GRE; after completing the program, each student takes another practice exam. Based on their performance, was the program effective?

- **Each subject contributes 2 scores: repeated measures design**

Student	Before Program	After Program
1	520	555
2	490	510
3	600	585
4	620	645
5	580	630
6	560	550
7	610	645
8	480	520

- **Can represent each student with a single score: the difference (D) between the scores**

Student	Before Program	After Program	D
1	520	555	35
2	490	510	20
3	600	585	-15
4	620	645	25
5	580	630	50
6	560	550	-10
7	610	645	35
8	480	520	40

- **Approach: test the effectiveness of program by testing significance of D**
- **Null hypothesis: There is no difference in the scores of before and after program**
- **Alternative hypothesis: program is effective → scores after program will be higher than scores before program → average D will be greater than zero**

$$H_0: \mu_D = 0$$
$$H_1: \mu_D > 0$$

So, need to know $\sum D$ and $\sum D^2$:

Student	Before Program	After Program	D	D^2
1	520	555	35	1225
2	490	510	20	400
3	600	585	-15	225
4	620	645	25	625
5	580	630	50	2500
6	560	550	-10	100
7	610	645	35	1225
8	480	520	40	1600
			$\sum D = 180$	$\sum D^2 = 7900$

Recall that for single samples:

$$t_{obt} = \frac{\overline{X} - \mu}{s_{\overline{X}}} = \frac{\text{score - mean}}{\text{standard error}}$$

For related samples:

$$t_{obt} = \frac{\overline{D} - \mu_D}{s_{\overline{D}}}$$

where:

$$s_{\overline{D}} = \frac{s_D}{\sqrt{N}}$$ and $$s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{N}}{N - 1}}$$

Mean of D:

$$\overline{D} = \frac{\sum D}{N} = \frac{180}{8} = 22.5$$

Standard deviation of D:

$$s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{N}}{N-1}} = \sqrt{\frac{7900 - \frac{(180)^2}{8}}{8-1}} = 23.45$$

Standard error:

$$s_{\overline{D}} = \frac{s_D}{\sqrt{N}} = \frac{23.45}{\sqrt{8}} = 8.2908$$

$$t_{obt} = \frac{\overline{D} - \mu_D}{s_{\overline{D}}}$$

Under H_0, $\mu_D = 0$, so:

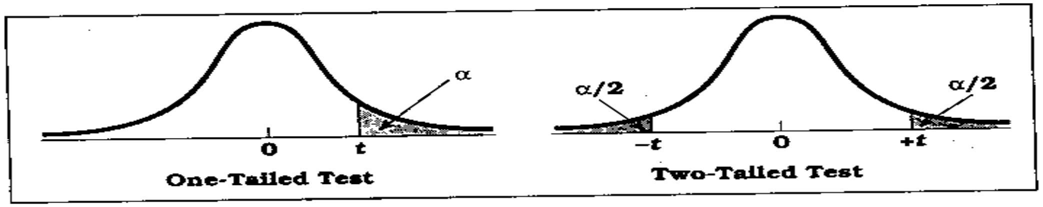
$$t_{obt} = \frac{\overline{D}}{s_{\overline{D}}} = \frac{22.5}{8.2908} = 2.714$$

From Table B.2: for $\alpha = 0.05$, one-tailed, with df = 7,

$t_{critical} = 1.895$

$2.714 > 1.895 \rightarrow$ reject H_0

The program is effective.



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	.25	.20	.15	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Z- value & t-Value

"Z and t" are the measures of:
How difficult is it to believe the null hypothesis?

High z & t values
Difficult to believe the null hypothesis - accept that there is a real difference.

Low z & t values
Easy to believe the null hypothesis - have not proved any difference.

Karl Pearson Correlation Coefficient

Working with two variables (parameter)

As Age ↑ BP ↑

As Height ↑ Weight ↑

As Age ↑ Cholesterol ↑

As duration ↑
of HIV CD4 CD8 ↓

A number called the **correlation** measures both the direction and strength of the linear relationship between two related sets of quantitative variables.

Correlation Contd….

- Types of correlation –
- Positive – Variables move in the same direction

- Examples:
- Height and Weight
- Age and BP

Correlation contd…

- Negative Correlation
- Variables move in opposite direction

- Examples:
- Duration of HIV/AIDS and CD4 CD8
- Price and Demand
- Sales and advertisement expenditure

Correlation contd…..

- Measurement of correlation

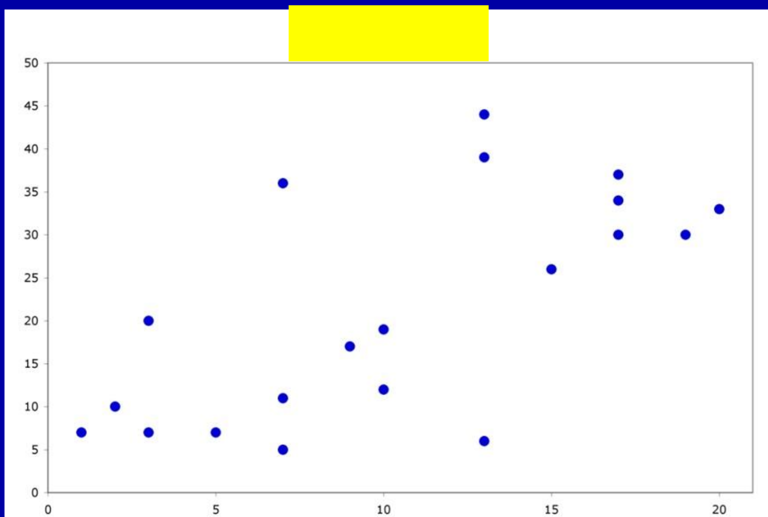
1. **Scatter Diagram**
2. **Karl Pearson's coefficient of Correlation**

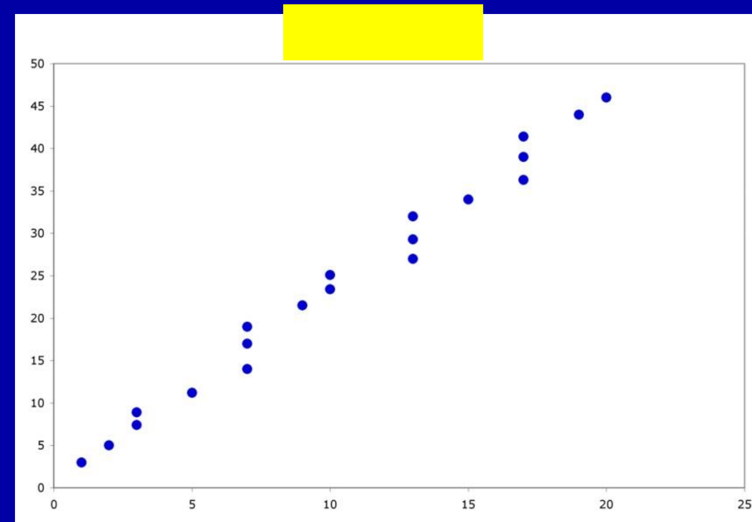
Graphical Display of Relationship

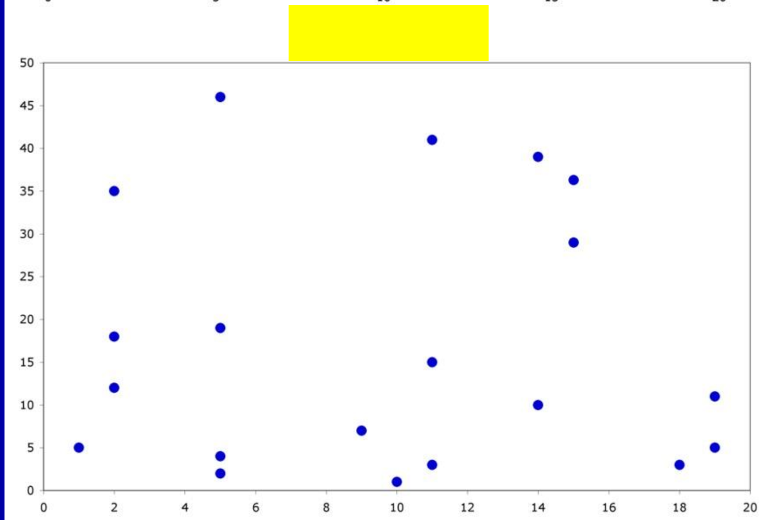
- Scatter diagram
- Using the axes
  - X-axis horizontally
  - Y-axis vertically
  - Both axes meet: origin of graph: 0/0
  - Both axes can have different units of measurement
  - Numbers on graph are (x,y)

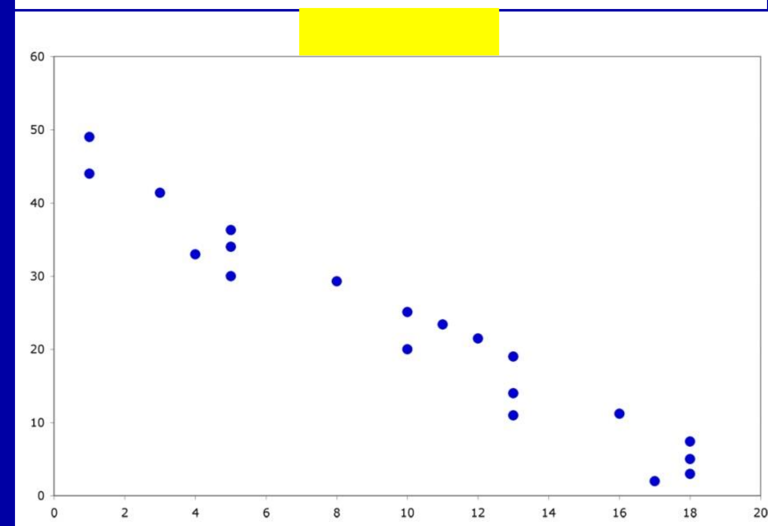
Guess the Correlations:

.67 .993 .003 -.975

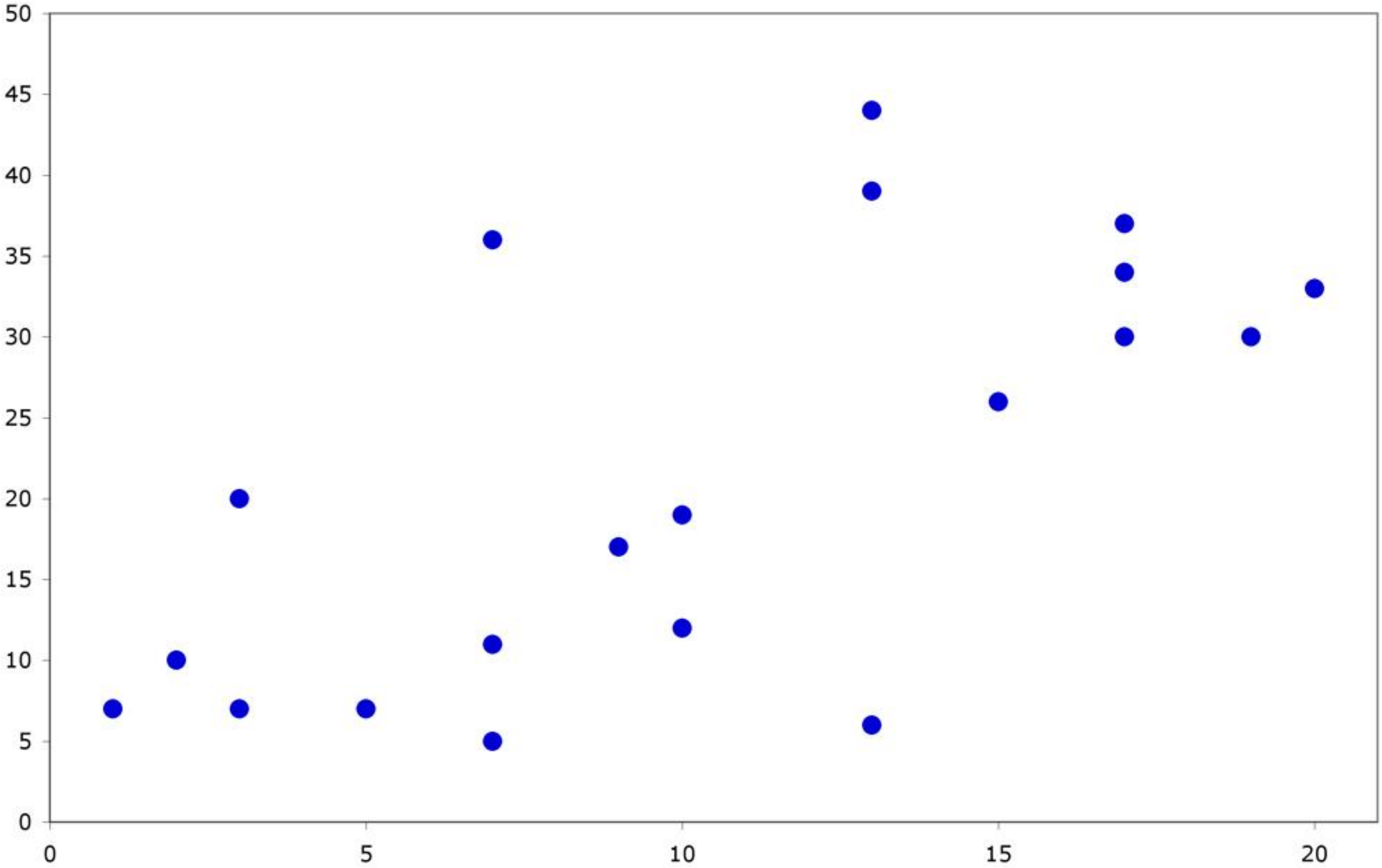




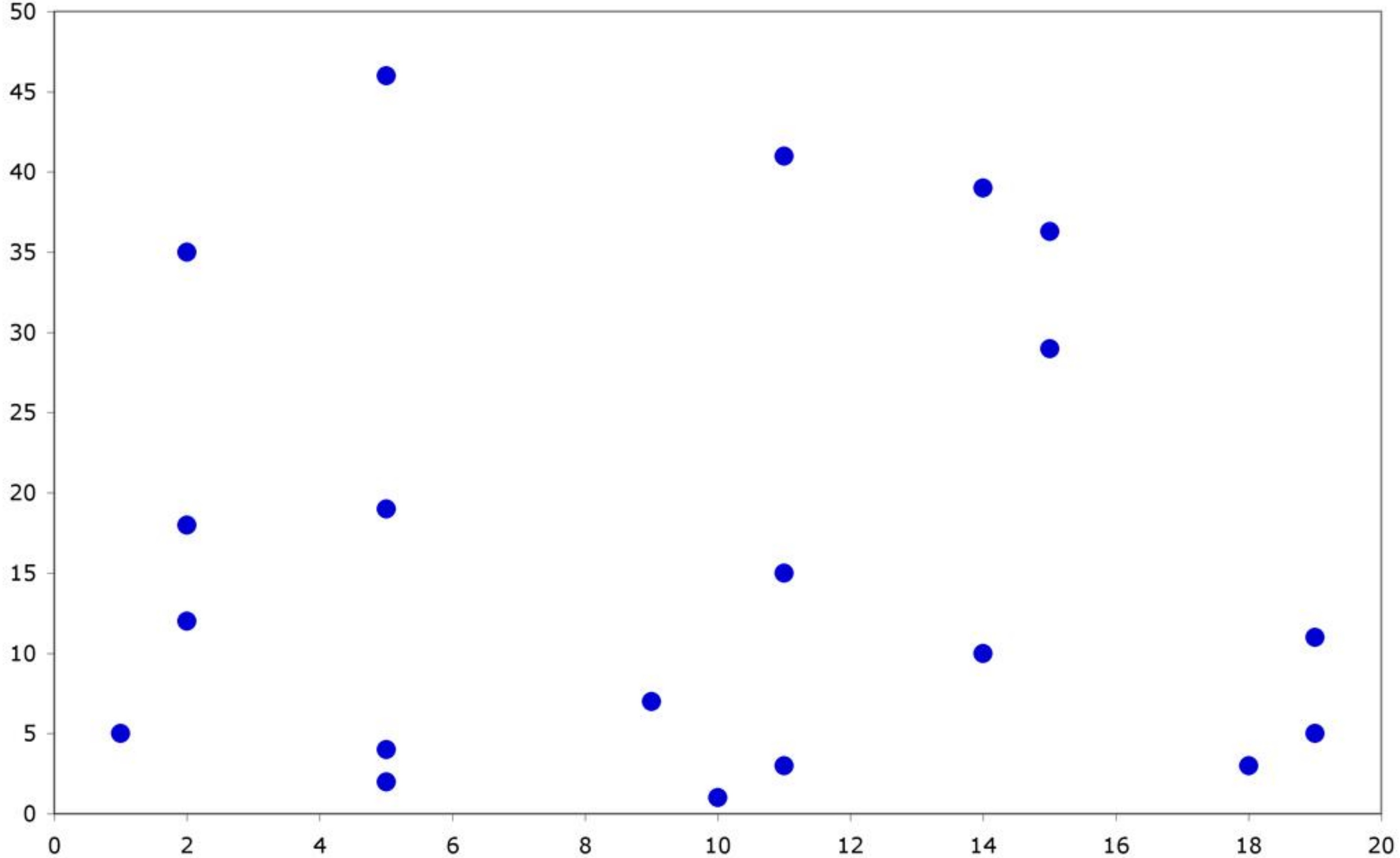




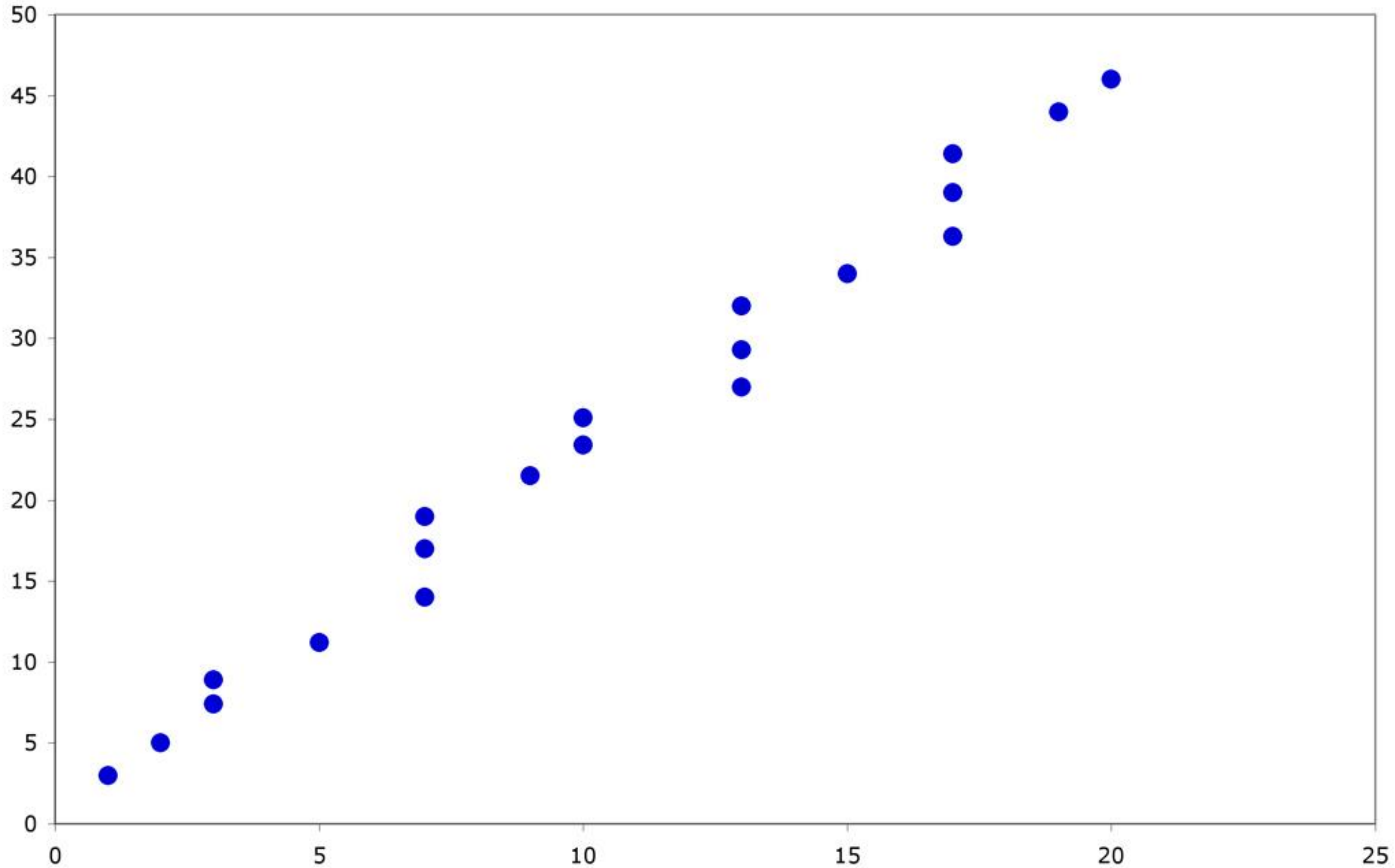
Correlation = .67

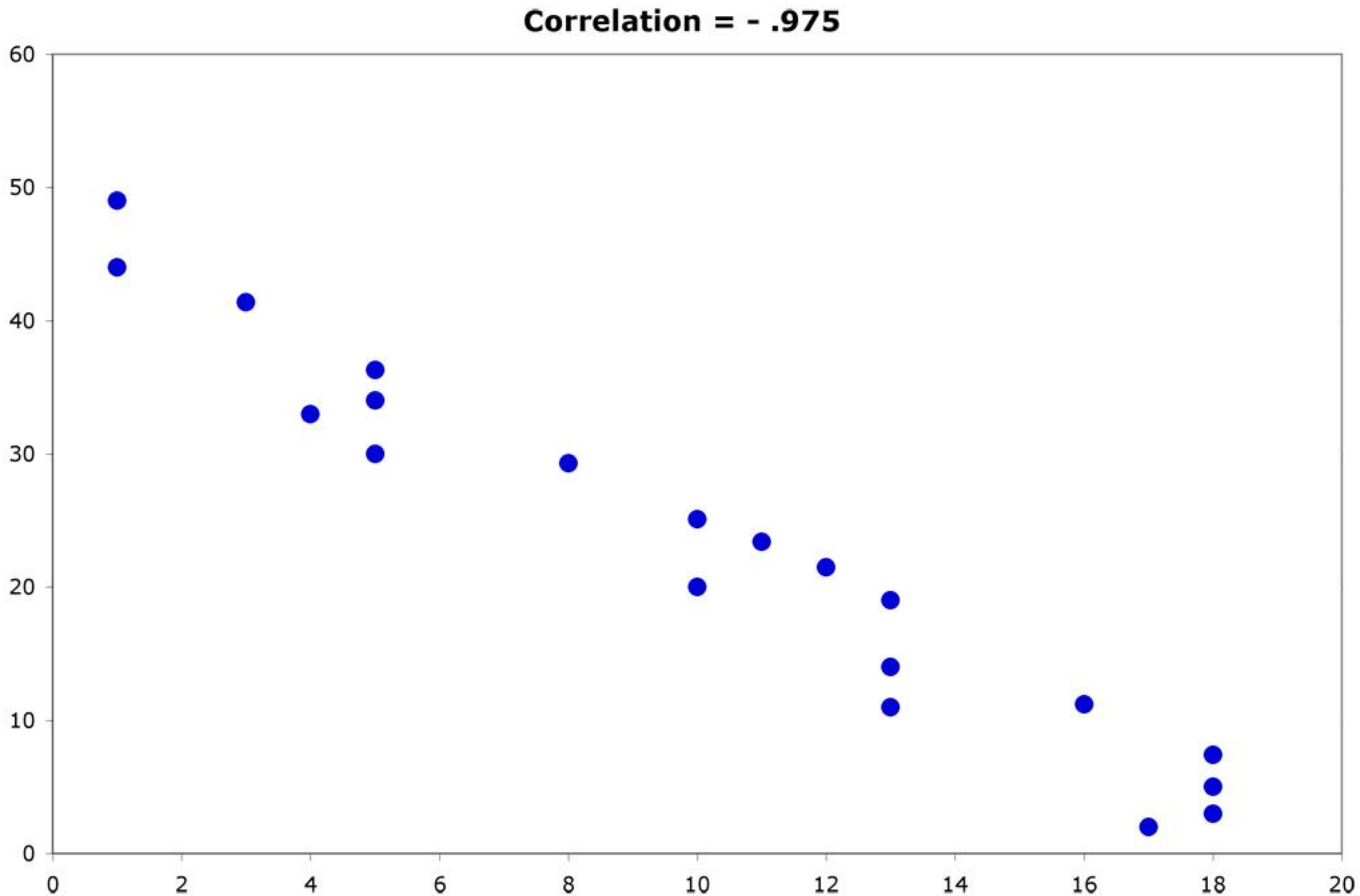


Correlation = .003



Correlation = .993




Correlation = - .975
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20

The Pearson r

$$r = \frac{\Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{N}}{\sqrt{\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]\left[\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right]}}$$

We Need:

- ***Sum of the Xs*** ΣX
- ***Sum of the Ys*** ΣY
- ***Sum of the Xs squared*** $(\Sigma X)^2$
- ***Sum of the Ys squared*** $(\Sigma Y)^2$
- ***Sum of the squared Xs*** ΣX^2
- ***Sum of the squared Ys*** ΣY^2
- ***Sum of Xs times the Ys*** ΣXY
- ***Number of Subjects*** (N)

Example:

A sample of 6 children was selected, data about their age in years and weight in kilograms was recorded as shown in the following table . Find the correlation between age and weight.

serial No	Age (years)	Weight (Kg)
1	7	12
2	6	8
3	8	12
4	5	10
5	6	11
6	9	13

Serial n.	Age (years) (x)	Weight (Kg) (y)	xy	X^2	Y^2
1	7	12	84	49	144
2	6	8	48	36	64
3	8	12	96	64	144
4	5	10	50	25	100
5	6	11	66	36	121
6	9	13	117	81	169
Total	$\sum x=$ 41	$\sum y=$ 66	$\sum xy=$ 461	$\sum x2=$ 291	$\sum y2=$ 742

$$r = \frac{461 - \frac{41 \times 66}{6}}{\sqrt{\left[291 - \frac{(41)^2}{6}\right] . \left[742 - \frac{(66)^2}{6}\right]}}$$

r = 0.759

strong direct correlation

EXAMPLE: Relationship between Anxiety and Test Scores

Anxiety (X)	Test score (Y)	X^2	Y^2	XY
10	2	100	4	20
8	3	64	9	24
2	9	4	81	18
1	7	1	49	7
5	6	25	36	30
6	5	36	25	30
$\sum X = 32$	$\sum Y = 32$	$\sum X^2 = 230$	$\sum Y^2 = 204$	$\sum XY=129$

Calculating Correlation Coefficient

$$r = \frac{(6)(129)-(32)(32)}{\sqrt{\left(6(230)-32^2\right)\left(6(204)-32^2\right)}} = \frac{774-1024}{\sqrt{(356)(200)}} = -.94$$

r = - 0.94

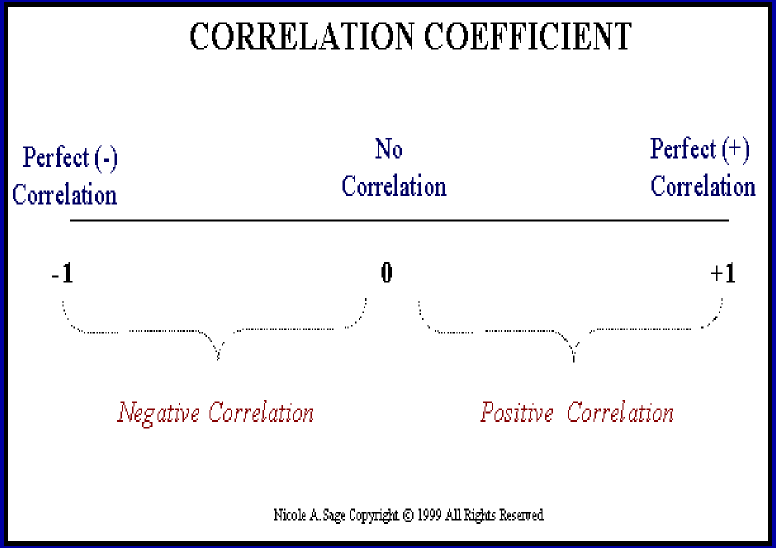
Indirect strong correlation

Correlation Coefficient

a correlation coefficient (r) provides a quantitative way to express the degree of linear relationship between two variables.

- Range: r is always between -1 and 1
- Sign of correlation indicates direction:
  - high with high and low with low -> positive
  - high with low and low with high -> negative
  - no consistent pattern -> near zero

- Magnitude (absolute value) indicates strength (-.9 is just as strong as .9)
  - .10 to .40 weak
  - .40 to .80 moderate
  - .80 to .99 high
  - 1.00 perfect



About “r”

- r is not dependent on the units in the problem
- r ignores the distinction between explanatory and response variables
- r is not designed to measure the strength of relationships that are not approximately straight line
- r can be strongly influenced by outliers

Correlation Coefficient: Limitations

1. Correlation coefficient is appropriate measure of relation <u>only when relationship is linear</u>
2. Correlation coefficient is appropriate measure of relation when equal ranges of scores in the sample and in the population.
3. Correlation <u>doesn't imply causality</u>
   - Using U.S. cities a cases, there is a strong positive correlation between the number of churches and the incidence of violent crime
   - Does this mean churches cause violent crime, or violent crime causes more churches to be built?
   - More likely, both related to population of city (3d variable -- lurking or confounding variable)

Ice-cream sales are strongly correlated with crime rates.

Therefore, ice-cream causes crime.

Without proper interpretation, causation **should not** be assumed, or even implied.

In conclusion !

Z-test will be used for both categorical(qualitative) and quantitative outcome variables.

Student's t-test will be used for only quantitative outcome variables.

Correlation will be used to quantify the linear relationship between two quantitative variables