Problem:

Assume that among diabetics the fasting blood level of glucose is approximately normally distributed with a mean of 105mg per 100ml and an SD of 9 mg per 100 ml. What proportion of diabetics having fasting blood glucose levels between 90 and 125 mg per 100 ml ?

NORMAL DISTRIBUTION AND ITS **APPL ICATION**

Objectives of this session:

 Able to understand the concept of Normal distribution.
 Able to calculate the z-score for quantitative variable.
 Able to apply the concept in the interpretation of a clinical data. The Normal or Gaussian distribution is the most important continuous probability distribution in statistics.

The term "Gaussian" refers to 'Carl Freidrich Gauss' who develop this distribution.

The word 'normal' here does not mean 'ordinary' or 'common' nor does it mean 'disease-free'.

It simply means that the distribution confirms to a certain formula and shape.

Gaussian Distribution

Many biologic variables follow this pattern

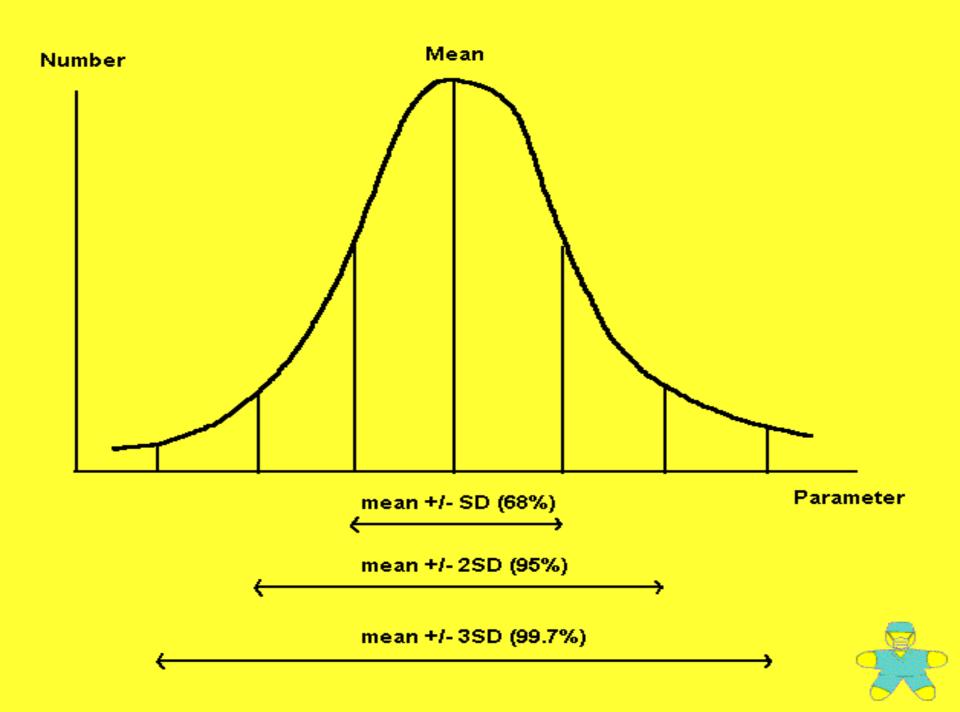
- Hemoglobin, Cholesterol, Serum Electrolytes, Blood pressures, age, weight, height
- One can use this information to define what is normal and what is extreme
- In clinical medicine 95% or 2 Standard deviations around the mean is normal
 - Clinically, 5% of "normal" individuals are labeled as extreme/abnormal
 - We just accept this and move on.

Height inches	No. of men of given height
61-62 62-63 63-64 64-65 65-66 66-67 67-68 68-69 69-70 70-71 71-72 72-73	$ \begin{array}{c} \frac{2}{5} \\ 17 \\ 43 \\ 86 \\ 152 \\ 193 \\ 197 \\ 148 \\ 91 \\ 45 \\ 16 \end{array} - 690 - 955 - 997 $
73–74 74–75	<u>4</u> 1
Total	1000
150 155	$\sigma = 6.5 \text{ cm}$ $\mu = 171.5 \text{ cm}$ $\mu = 171.5 \text{ cm}$ $160 165 170 175 180 185 190$ Height (cm)
	-2 -1 0 1 2 3 SND

2

Table 9.3 Example of a Normal Distribution—Distribution of 1000 Men in a Village According to Their Height

Fig. 5.2 Relationship between normal distribution in original units of measurement and in standard normal deviates. SND = (height - 171.5)/6.5. $Height = 171.5 + (6.5 \times SND)$.



Characteristics of Normal Distribution Symmetrical about mean, μ Mean, median, and mode are equal Total area under the curve above the xaxis is one square unit 1 standard deviation on both sides of the mean includes approximately 68% of the total area 2 standard deviations includes approximately 95% 3 standard deviations includes

approximately 99%

Uses of Normal Distribution

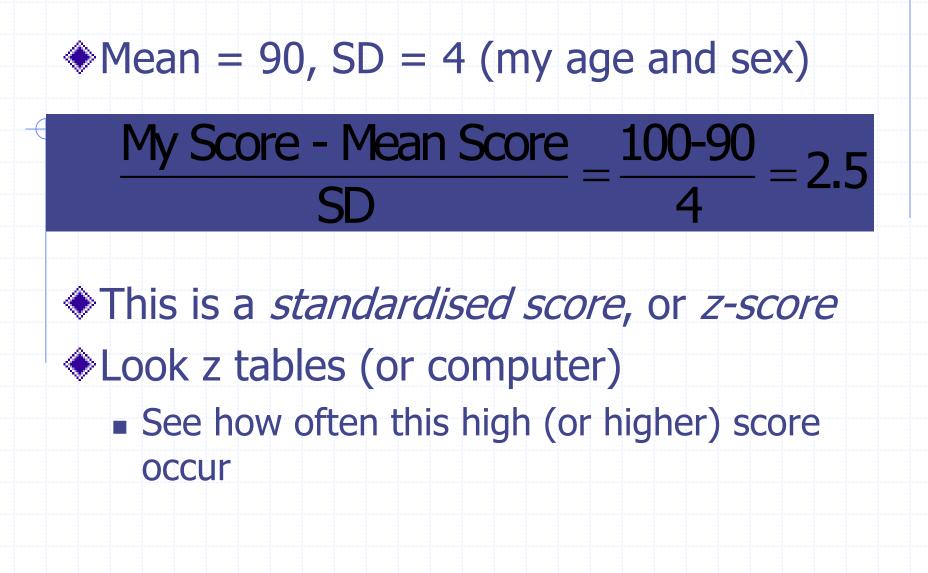
- It's application goes beyond describing distributions
- It is used by researchers.
- The major use of normal distribution is the role it plays in statistical inference.

It helps managers to make decisions.

What's so Great about the Normal Distribution?

If you know two things, Mean Standard deviation you know everything about the distribution You know the probability of any value arising

Standardised Scores My diastolic blood pressure is 100 So what ? Normal is 90 (for my age and sex) Mine is high • But how much high? Express it in standardised scores How many SDs above the mean is that?



Measures of Position

Z Score (or standard score)

the number of standard deviations that a given value *x* is above or below the mean

Standard Scores

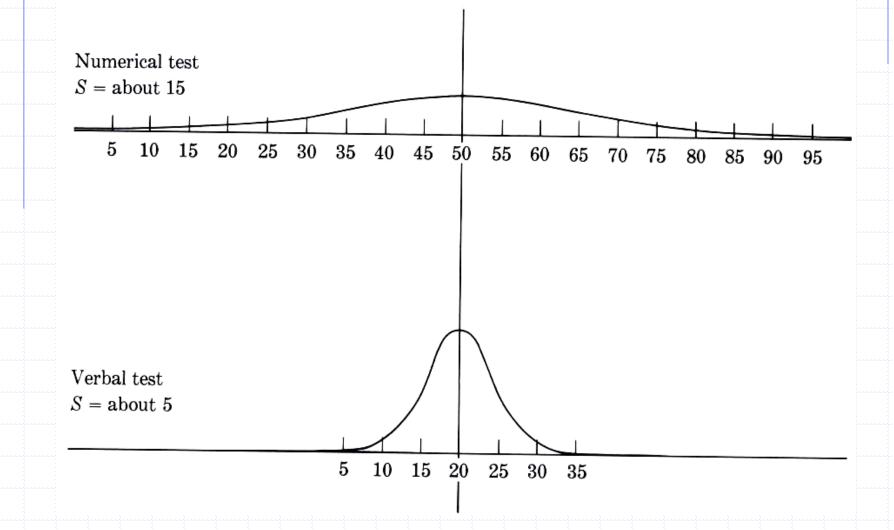
The Z score makes it possible, under some circumstances, to compare scores that originally had different units of measurement.

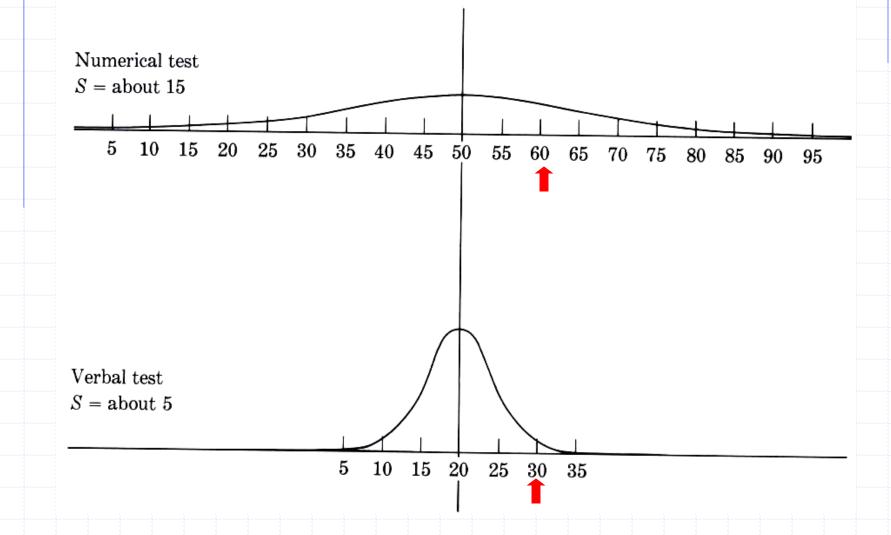
Suppose you scored a 60 on a numerical test and a 30 on a verbal test. On which test did you perform better?

- First, we need to know how other people did on the same tests.
 - Suppose that the mean score on the numerical test was 50 and the mean score on the verbal test was 20.
 - You scored 10 points above the mean on each test.
 - Can you conclude that you did equally well on both tests?
 - You do not know, because you do not know if 10 points on the numerical test is the same as 10 points on the verbal test.

Suppose you scored a 60 on a numerical test and a 30 on a verbal test. On which test did you perform better?

- Suppose also that the standard deviation on the numerical test was 15 and the standard deviation on the verbal test was 5.
 - Now can you determine on which test you did better?

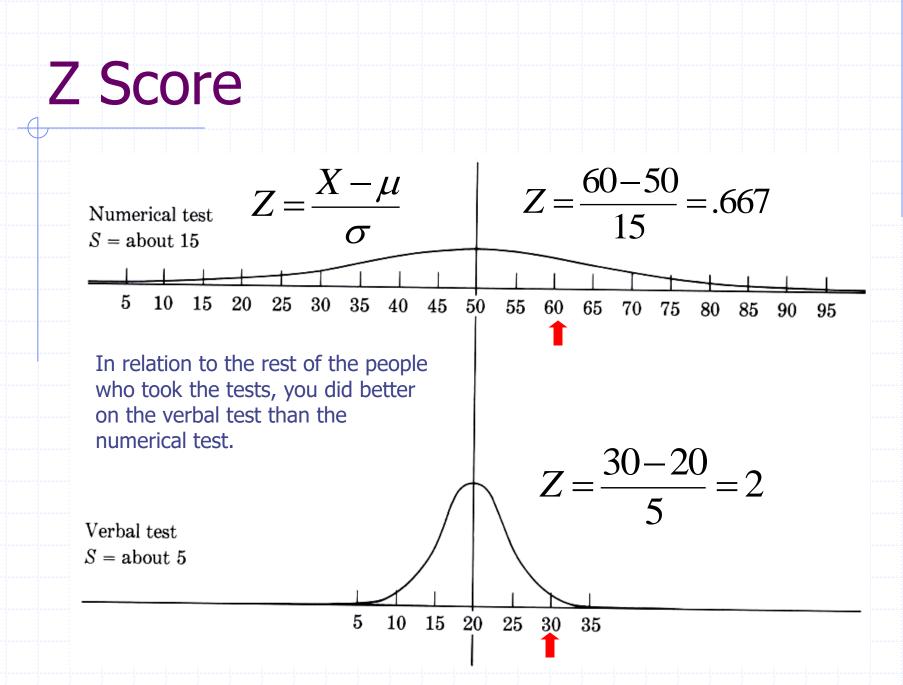




Z score

 To find out how many standard deviations away from the mean a particular score is, use the Z formula: Population: Sample:

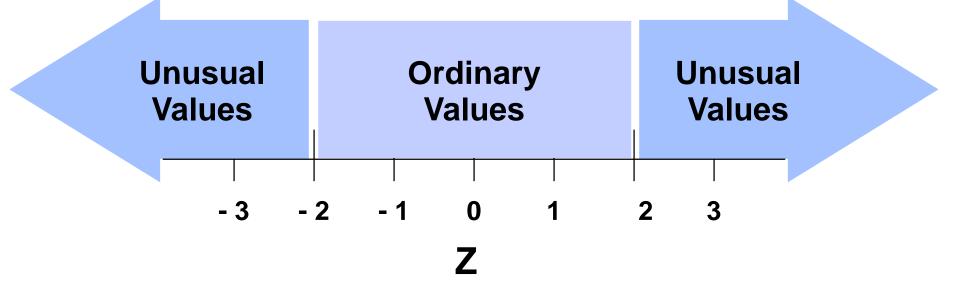




Z score

- Allows you to describe a particular score in terms of where it fits into the overall group of scores.
 - Whether it is above or below the average and how much it is above or below the average.
- A standard score that states the position of a score in relation to the mean of the distribution, using the standard deviation as the unit of measurement.
 - The number of standard deviations a score is above or below a mean.

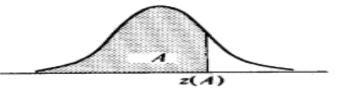
Interpreting Z Scores



The Standard Normal Table

Using the standard normal table, you can find the area under the curve that corresponds with certain scores. The area under the curve is proportional to the frequency of scores. The area under the curve gives the probability of that score occurring.

Entry is area A under the standard normal curve from $-\infty$ to z(A)



						_				
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.8 .9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
	.0157	.0100	.0212	10250	.0201	.0207				
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.6	.9965	.9955	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.7	.9905	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.8		.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
2.9	.9981	.9982	.9962	.9903	.7704	.7704	.9905		.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
	1									

Standard Normal Table

11.5 11.5 <th< th=""><th>A</th><th>в</th><th>с</th><th>A</th><th>в</th><th>С</th><th>A</th><th>в</th><th>С</th></th<>	A	в	с	A	в	С	A	в	С
100 4546 0465 2.25 4478 0122 2.81 44775 0023 170 4554 0446 2.26 4484 1016 2.82 44975 0023 171 4554 0446 2.27 4484 1016 2.82 44977 0023 173 4552 0418 2.28 4480 0110 2.85 44979 0021 174 4551 0401 2.31 44968 0104 2.86 4979 0021 175 4666 03344 2.33 44961 0099 2.90 4481 0017 176 4606 03357 2.35 44904 0099 2.90 4481 0017 178 4625 0357 2.35 44906 0091 2.92 4482 0018 180 4641 0356 2.38 4916 0084 2.95 44863 0016 181 4666 0314 2.4	z	\mathbb{A}	Δ	z =	\wedge		z =	\wedge	\wedge
170 4554 0446 2.26 4881 0119 2.82 4976 0024 171 4564 0465 2.27 4887 0113 2.84 4977 0023 172 4573 0427 2.28 4887 0113 2.84 4977 0023 173 4552 0418 2.29 4893 0107 2.85 4979 0021 174 4591 0409 2.30 4896 0104 2.87 4979 0021 176 4599 0401 2.31 4896 0104 2.87 4979 0021 176 6608 0332 2.32 4896 01099 2.89 4981 0019 178 4625 0375 2.34 4906 0096 2.90 4982 0018 180 44641 0359 2.35 4906 0094 2.91 4982 0016 181 44649 0351 2.37	1.68								
173 4582 0418 2.29 4480 0110 2.85 4478 0022 175 4599 0401 2.31 4486 0104 2.86 4979 0021 176 4608 0352 2.32 4486 0104 2.87 4481 0072 177 4616 0354 2.33 4491 0099 2.88 4490 0091 178 4625 0357 2.35 44904 0096 2.90 4891 0019 179 4633 0351 2.37 4911 0087 2.94 4892 0018 181 4649 0351 2.37 4911 0087 2.94 4893 0017 182 4656 0354 2.38 4916 0084 2.95 4893 0016 184 4671 0322 2.41 4920 0080 2.97 4894 0061 184 4664 0322 2.44	1.70	.4554 .4564	.0446 .0436	2.26 2.27	.4881 .4884	.0119 .0116	2.82 2.83	.4976	.0024 .0023
175 4599 0401 2.31 4486 0104 2.47 44979 0020 17.7 6408 0354 2.32 4486 00102 2.48 44901 0009 17.7 4616 0354 2.33 44901 00096 2.48 44901 0019 17.8 4625 0357 2.35 44906 00964 2.90 4891 0019 17.9 4633 0357 2.35 44906 0091 2.92 44982 0018 18.1 4644 0355 2.37 4911 0087 2.94 4983 0017 18.2 4656 03341 2.42 4920 0080 2.97 44985 0015 18.4 4671 0322 2.41 4920 00075 2.98 4986 0014 18.4 4693 03017 2.44 4925 0075 2.98 4986 0011 18.4 4769 02017	1.73	.4582	.0418	2.29	.4890	.0110	2.85	.4978	.0022
177 4616 0384 2.33 4901 0.099 2.89 4981 0.019 178 4625 0375 2.34 4904 0096 2.90 4981 0019 179 4623 0387 2.35 4406 0094 2.91 4982 0018 180 4644 0355 2.35 4406 0097 2.94 4982 0018 181 4649 0355 2.38 44913 0087 2.94 4983 0017 182 4656 0344 2.38 44916 0084 2.95 4984 0016 183 4664 0322 2.41 4920 0080 2.97 4985 0015 186 4686 0314 2.42 4922 0075 2.98 4986 0014 187 4693 0090 3.02 4987 0013 3.04 4987 0013 191 4776 0274 2.45	1.74	.4599	.0401	2.31	.4896	.0104	2.87	.4979	.0021
178 4625 0.0375 2.34 4404 0.006 2.40 4491 0.011 179 4633 0357 2.35 44096 0.0011 2.42 44982 0018 180 4641 03551 2.37 44911 0.0089 2.43 44982 0018 181 4644 03551 2.37 44911 0.0087 2.44 44982 0.0016 183 4664 03344 2.38 44913 0.0087 2.45 44984 0.0016 184 4664 03252 2.401 44720 0.0080 2.47 44985 0.0015 186 46663 03017 2.44 44727 0.073 3.00 4986 0.0014 187 4663 03017 2.44 44727 0.073 3.00 4987 0013 188 4698 0.0301 2.47 44932 0.068 3.03 4987 0013 1912 4776	1.76								
100 4411 0059 2.36 44009 0.091 2.42 4482 0.011 181 4449 0351 2.37 4411 0.089 2.33 4483 0.017 182 44654 0.0354 2.38 44913 0.007 2.44 4484 0.016 184 4671 0.329 2.40 44918 0.0082 2.95 44945 0.0075 185 4673 0.322 2.41 44922 0.0075 2.99 4996 0.014 186 4686 0.0314 2.42 4922 0.077 2.49 4996 0.014 187 4693 0.007 2.44 4727 0.073 3.00 4987 0.013 188 4706 0.0274 2.45 44932 0.068 3.03 4987 0.011 193 4774 0.288 2.409 4934 0.066 3.04 4984 0.011 194 4784 <	1.78	.4625	.0375	2.34	.4904	.0096	2.90	.4981	.0019
1 # 4449 0351 2.37 4911 0089 2.43 4493 0017 1 82 4656 0344 2.38 44913 0087 2.44 4984 0016 1 84 4671 0325 2.38 44916 0084 2.95 4984 0016 1 84 6671 0322 2.41 4920 0080 2.97 4985 0015 1 85 4668 0314 2.42 4922 0075 2.98 4986 0014 1 87 7.464 4925 0075 2.99 4986 0013 1 89 4706 0294 2.44 4925 0071 3.00 4987 0013 1 90 4713 0287 2.46 4929 0071 3.01 4987 0013 1 91 4776 0291 2.47 4932 0066 3.02 4987 0013 1 92 4726 0274 2.48 4932 00	1.79	.4633	.0367						
13 4664 0336 2.39 4916 0084 2.95 4984 0016 184 4678 0322 2.40 4418 0082 2.96 4985 0015 185 4678 0322 2.41 4420 0080 2.97 4985 0015 186 6686 0314 2.42 4422 2.077 2.98 4986 0014 187 4693 0.007 2.43 4425 0.075 2.98 4986 0.014 188 4699 0.001 2.44 4427 0.077 3.00 4987 0.013 190 4773 0.287 2.46 4932 0.0669 3.02 4987 0.013 191 4773 0.287 2.46 4932 0.0669 3.02 4987 0.013 192 4776 0.274 2.448 4936 0.066 3.04 4988 0.011 193 4775 0.265	1.81	.4649	.0351	2.37	.4911	.0089	2.93	.4983	.0017
14 4671 0329 240 4918 0.002 2.96 4985 0015 1.85 4676 0322 2.41 4920 0.080 2.97 4985 0015 1.86 4686 0.0314 2.42 4922 0.078 2.98 4986 0014 1.87 4685 0.0301 2.44 4727 0.073 3.00 4987 0013 1.88 4706 0.291 2.44 4727 0.073 3.01 4987 0013 1.90 4713 0.287 2.45 44931 0.068 3.03 4988 0.012 1.91 4719 0.287 2.46 4934 0.066 3.04 4988 0.011 1.92 4726 0.0274 2.49 4934 0.066 3.04 4988 0.011 1.94 4778 0.0250 2.52 4944 0.057 3.09 4989 0.011 1.96 4774 0.02	1.82								
186 4886 0314 2.42 4922 0078 2.88 4986 0014 187 4683 0307 2.43 4922 0078 2.99 4986 0014 188 4669 0301 2.44 4727 0075 3.00 4987 0013 189 4766 0287 2.45 4491 0068 3.02 4987 0013 180 4773 0287 2.45 4493 0068 3.03 4987 0013 181 4773 0287 2.46 4934 00668 3.04 4988 0012 182 4734 02852 2.50 4944 00660 3.05 4989 0011 194 4734 02550 2.51 4944 0055 3.08 4980 0010 196 4766 02544 2.53 4945 0055 3.10 4980 0011 197 4756 02244 2.53 <td>1.84</td> <td>.4671</td> <td>.0329</td> <td>2.40</td> <td>.4918</td> <td>.0082</td> <td>2.96</td> <td>.4985</td> <td>.0015</td>	1.84	.4671	.0329	2.40	.4918	.0082	2.96	.4985	.0015
187 4693 0.007 2.43 4425 0.075 2.99 4.986 0.011 188 4696 0.001 2.44 4727 0.073 3.00 4987 0.013 189 4706 0.294 2.45 4429 0.0071 3.01 4987 0.013 190 4713 0.0281 2.45 44329 0.0069 3.02 4987 0.013 191 4719 0.0281 2.47 44932 0.0669 3.04 4988 0.012 192 4726 0.0214 2.48 4932 0.0669 3.04 4988 0.011 194 4738 0.0256 2.51 4940 0.0609 3.06 4989 0.011 195 4744 0.256 2.51 4944 0.0655 3.10 4990 0.010 198 4767 0.233 2.55 4944 0.055 3.10 4991 0.009 2.04 4733									
189 4706 10294 2.45 4929 10071 3.01 4987 0.013 190 4713 10287 2.46 4931 10069 3.02 4987 0.013 191 4719 10281 2.47 44932 10069 3.02 4987 0.013 192 4726 10274 2.48 4932 10066 3.04 4988 0.012 133 4732 10288 2.48 4936 0.064 3.05 4989 0.011 134 4738 0265 2.51 4940 0.069 3.07 4989 0.011 136 4776 0256 2.51 4940 0.069 3.08 4989 0.011 136 4776 0224 2.53 4943 0.065 3.10 4990 0.010 198 4767 0233 2.55 4946 0.054 3.11 4991 0.009 2.04 4778 0222	1.87	.4693	.0307	2.43	.4925	.0075			
100 4713 10287 2.46 4931 0.0069 3.02 4.4887 0.0013 191 4719 10281 2.47 44932 0.0068 3.03 44887 0.0012 192 4726 0.0274 2.48 44934 0.0066 3.04 44888 0.0012 193 47726 0.0284 2.49 44936 0.0666 3.05 44989 0.0011 194 4738 0.0250 2.51 4940 0.060 3.07 4899 0.0011 196 4776 0.0250 2.52 4941 0.059 3.08 4990 0.0010 197 4776 0.0250 2.55 4944 0.055 3.10 4990 0.0010 198 4761 0.0233 2.55 4944 0.055 3.11 4991 0.009 2.01 4777 0.0228 2.55 4944 0.051 3.13 4991 0.009 2.02 2.									
1.1 2.48 4.4934 0.0665 3.04 4.4888 0.0011 1.83 4.736 0.2668 2.49 4.4934 0.0666 3.04 4.4888 0.0011 1.83 4.732 0.2668 2.49 4.4938 0.062 3.05 4.4989 0.0111 1.94 4.734 0.2622 2.51 4.4940 0.0660 3.07 4.899 0.0111 1.96 4.750 0.2250 2.52 4.4941 0.0557 3.08 4.990 0.010 1.97 4.756 0.2244 2.53 4.4945 0.0055 3.10 4.990 0.010 1.98 4.767 0.233 2.55 4.4945 0.0055 3.11 4.990 0.010 1.99 4.767 0.233 2.55 4.4945 0.0052 3.13 4.991 0.0005 2.01 4.772 0.223 2.56 4.4951 0.0049 3.14 4.992 0.006 2.020 4.4733 <td>1.90</td> <td>.4713</td> <td>.0287</td> <td>2.46</td> <td>.4931</td> <td>.0069</td> <td>3.02</td> <td></td> <td></td>	1.90	.4713	.0287	2.46	.4931	.0069	3.02		
194 4738 0.052 2.50 4938 0.062 3.06 4989 0.011 195 4744 0.255 2.51 4940 0.060 3.07 4989 0.011 196 47744 0.255 2.52 4941 0.057 3.09 4990 0.010 197 4756 0.0244 2.53 4943 0.057 3.09 4990 0.010 198 4771 0.233 2.54 4945 0.055 3.10 4990 0.010 199 4772 0.238 2.55 4946 0.051 3.13 4991 0.009 2.00 4778 0.222 2.57 4949 0.051 3.13 4991 0.009 2.03 4778 0.0217 2.58 4951 0.049 3.14 4992 0.008 2.03 4788 0.0217 2.58 4955 0.044 3.18 4992 0.008 2.03 4783 0	1.91			2.47					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1.93								
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1.39 1.761 0.254 1.445 0.055 3.10 0900 0010 1.98 1.761 0.239 2.54 4.945 0.065 3.11 4.990 0010 1.99 1.767 0.233 2.55 4.946 0.0652 3.11 4.991 0000 2.01 1.777 0.223 2.57 4.946 0.0652 3.13 4.991 0000 2.01 4.778 0.227 2.57 4.9651 0.049 3.14 4.991 0.000 2.02 4.783 0.227 2.58 4.9651 0.049 3.15 4.992 0.008 2.03 4.778 0.2207 2.40 4.953 0.047 3.16 4.992 0.008 2.06 4.953 0.0477 3.16 4.992 0.008 0.077 2.00 4.933 0.007 2.06 4.955 0.0445 3.18 4.943 0.007 2.09 4.933 0.007 2.046 4.965 <td>1.96</td> <td>.4750</td> <td>.0250</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	1.96	.4750	.0250						
199 4767 0.0233 2.55 4946 0.0054 3.11 4991 0.009 2.00 4772 0.228 2.56 4946 0.0654 3.112 4991 0.009 2.01 4778 0.022 2.57 4949 0.0051 3.13 4.991 0.009 2.01 4778 0.0217 2.58 4951 0.0049 3.14 4992 0.000 2.03 4786 0.212 2.59 4952 0.0048 3.15 4992 0.000 2.04 4733 0.2072 2.60 4955 0.0047 3.16 4992 0.000 2.05 4495 0.0042 3.17 4993 0.007 2.60 4955 0.0044 3.18 4992 0.000 2.06 4952 0.0456 3.17 4993 0.007 2.61 4965 0.044 3.18 4993 0.007 2.06 4817 0.188 2.64 4959 0.041			100000000000000000000000000000000000000				3.10	.4990	.0010
211 4778 1022 2.57 4949 0.051 3.13 4991 0.009 202 4788 0.0217 2.58 4949 0.061 3.14 4991 0.008 2.03 4788 0.0217 2.58 4951 0.0049 3.14 4992 0.008 2.04 4778 0.0207 2.59 4952 0.048 3.15 4992 0.008 2.06 4953 0.047 3.16 4992 0.008 2.06 4933 0.097 2.61 4955 0.045 3.17 4.992 0.008 2.06 4933 0.097 2.62 4956 0.044 3.18 4.993 0.007 2.07 4808 0.162 2.63 4967 0.043 3.19 4.993 0.007 2.08 4812 0.183 2.66 4960 0.040 3.22 4.993 0.007 2.10 4817 0.174 2.67 4962	1.99	.4767	.0233						
La. La. <thla.< th=""> <thla.< th=""> <thla.< th=""></thla.<></thla.<></thla.<>	2.01	.4778	.0222	2.57	.4949	.0051	3.13	.4991	.0009
2 04 + 4783 0.007 2 60 4953 0.047 3.16 4992 0.008 2 05 4798 0.007 2.61 4955 0.044 3.17 4992 0.008 2 06 4798 0.0197 2.62 44955 0.044 3.18 4993 0.000 2 06 4403 0.1197 2.62 44956 0.044 3.18 4993 0.007 2 08 44817 0.1188 2.64 4959 0.041 3.20 4993 0.007 2 08 44817 0.1188 2.64 4959 0.041 3.20 4993 0.007 2 08 4.812 0.1179 2.65 4961 0.039 3.22 4994 0.006 2 11 4.825 0.177 2.66 4962 0.039 3.23 4994 0.006 2 12 4.830 0.162 2.70 4965 0.035 3.30 4994 0.006 2 14 4.838 </td <td>2.02</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	2.02								
2 06 2 07 4803 4803 .0197 .0197 2 62 2 63 4956 4857 .0044 0.043 3.18 3.19 4983 4993 .0007 2.06 .4812 .0188 .263 .4957 .0044 .3.19 .4993 .0007 2.08 .4812 .0188 .264 .4957 .0044 .3.19 .4993 .0007 2.09 .4817 .0185 .265 .4951 .0041 .3.20 .4993 .0007 2.10 .4821 .0179 .2.457 .4961 .0039 .3.22 .4994 .0006 2.11 .4824 .0170 .2.68 .4963 .0037 .3.24 .4994 .0006 2.12 .4830 .0170 .2.68 .4963 .0035 .3.35 .4994 .0006 2.14 .4838 .0162 .2.70 .4965 .0035 .3.35 .4994 .0006 2.14 .4838 .0164 .2.72 .4965 .0033 .3.46 .4997 .0003	2.03			2.60	.4953	.0047	3.16	.4992	.0008
2.07 4806 .0192 2.63 4957 .0043 3.19 4993 .0007 2.08 .4812 .0188 2.64 .4959 .0041 3.20 .4993 .0007 2.09 .4817 .0183 2.65 .4960 .0040 3.21 .4993 .0007 2.10 .4821 .0179 2.66 .4961 .0039 3.22 .4994 .0006 2.11 .4826 .0177 2.66 .4961 .0039 3.22 .4994 .0006 2.12 .4830 .0170 2.68 .4963 .0037 3.24 .4994 .0006 2.13 .4834 .0166 2.69 .4964 .0036 .3.25 .4994 .0006 2.14 .4338 .0162 .2.71 .4966 .0033 .3.30 .4995 .0004 2.16 .4442 .0154 .2.72 .4967 .0033 .3.46 .4997 .0003 2.17	2.05			2.61					
209 4817 .0183 2.65 4960 .0040 3.21 4933 .0007 210 4821 .0179 2.66 .4961 .0039 3.22 .4933 .0007 2.11 .4826 .0174 2.67 .4962 .0038 3.23 .4993 .0006 2.12 .4830 .0177 2.68 .4962 .0038 3.23 .4994 .0006 2.12 .4834 .0166 2.69 .4964 .0035 3.30 .4994 .0006 2.14 .4838 .0166 2.69 .4964 .0036 3.25 .4994 .0006 2.14 .4838 .0156 2.70 .4965 .0035 .3.30 .4994 .0005 2.16 .4846 .0154 2.72 .4966 .0033 .3.40 .4996 .0003 2.17 .4865 .0032 .3.45 .4997 .0003 .4997 .0003 2.18 .4857	2.00	.4808	.0192		.4957	.0043	3.19	.4993	.0007
210 .4821 .0179 2.66 .4961 .0039 3.22 .4994 .0006 211 4480 .0170 2.66 .4961 .0039 3.22 .4994 .0006 212 .4830 .0170 2.68 .4962 .0038 3.23 .4994 .0006 213 .4834 .0166 2.69 .4964 .0036 3.25 .4994 .0006 214 .4838 .0162 2.70 .4965 .0035 3.35 .4995 .0005 2.16 .4446 .0154 2.77 .4966 .0032 3.46 .4997 .0033 2.16 .4464 .0154 2.77 .4966 .0032 3.45 .4997 .0033 2.17 .4954 .0146 2.74 .4969 .0031 .3.50 .4998 .0002 2.18 .4854 .0146 2.77 .4970 .0033 .3.60 .4998 .0001 .20 .4864	2.08								
212 4830 .0170 2.68 .4963 .0037 3.24 .4994 .0006 213 .4834 .0166 2.69 .4964 .0036 3.25 .4994 .0006 214 .4838 .0162 2.70 .4965 .0035 3.30 .4995 .0005 215 .4646 .0154 2.71 .4965 .0035 3.35 .4996 .0031 216 .4464 .0154 2.73 .4966 .0032 3.46 .4997 .0033 2.17 .4865 .0130 2.50 .4998 .0002 .345 .4997 .0033 2.16 .4466 .0154 2.73 .4966 .0032 .345 .4997 .0033 2.18 .4857 .0146 2.74 .4969 .0031 .3.50 .4998 .0002 2.20 .4861 .0139 2.76 .4971 .0029 .3.70 .4998 .0001 .21 .4864	2.10	.4821	.0179	2.66	.4961	.0039	3.22	.4994	.0006
2.13 .4834 .0166 2.69 .4964 .0036 3.25 .4994 .0005 2.14 .4838 .0162 2.70 .4965 .0035 3.30 .4995 .0005 2.15 .4842 .0158 2.71 .4965 .0033 .3.30 .4995 .0005 2.16 .4846 .0154 2.72 .4967 .0033 .3.40 .4997 .0003 2.17 .4850 .0150 2.73 .4968 .0032 .3.45 .4997 .0003 2.18 .4854 .0146 2.75 .4970 .0033 .3.60 .4998 .0002 2.19 .4857 .0143 2.75 .4971 .0029 .3.70 .4998 .0002 2.20 .4861 .0139 2.76 .4971 .0029 .3.70 .4999 .0011 2.21 .4868 .0132 .2.76 .4973 .0027 .3.90 .49995 .00005 2.22 </td <td>2.11</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	2.11								
215 4842 0158 2.71 4966 0.034 3.35 4996 0.004 216 4.846 0.154 2.72 4967 0.033 3.40 4996 0.003 2.16 4.846 0.1540 2.72 4967 0.033 3.40 4997 0.003 2.17 4.850 0.1500 2.73 4968 0.0032 3.45 4997 0.003 2.18 4.854 .0146 2.74 4.969 0.0031 3.50 4.998 0.002 2.19 4.851 .0139 2.75 4.970 0.0030 3.60 4.998 0.002 2.20 4.861 .0139 2.76 4.971 .0029 3.70 4.999 .001 2.21 .4864 .0136 2.77 4.972 .0029 3.70 4.999 .001 2.22 .4868 .0132 2.78 .4973 .0027 3.90 .49995 .00005	2.12	.4834	.0166	2.69	.4964	.0036	3.25	.4994	.0006
216 .4846 .0154 2.72 .4967 .0033 3.40 .4967 .0003 2.17 .4850 .0150 2.73 .4968 .0032 3.45 .4967 .0003 2.18 .4854 .0146 2.74 .4969 .0031 3.50 .4968 .0022 2.19 .4857 .0146 2.74 .4969 .0031 3.50 .4968 .0022 2.19 .4857 .0143 2.75 .4970 .0030 3.60 .4968 .0022 2.00 .4861 .0139 2.76 .4971 .0029 3.70 .4988 .0001 2.21 .4864 .0136 2.77 .4972 .0029 3.80 .4999 .0001 2.21 .4868 .0132 2.778 .4973 .0022 3.80 .4999 .0001 2.22 .4868 .0132 .278 .4973 .0022 .300 .49995 .00005	2.14						3.30 3.35		.0005
1.1 1.1 <th1.1< th=""> <th1.1< th=""> <th1.1< th=""></th1.1<></th1.1<></th1.1<>	2.16	.4846	.0154	2.72	.4967	.0033	3.40	.4997	.0003
2.19 4.857 .0143 2.75 .4970 .0030 3.60 .4998 .0002 2.20 .4661 .0139 2.76 .4971 .0029 3.70 .4999 .0001 2.21 .4864 .0136 2.77 .4972 .0028 3.80 .4999 .0001 2.22 .4868 .0132 2.78 .4973 .0027 3.90 .4995 .00005	2.17								
2.21 .4864 .0136 2.77 .4972 .0028 3.80 .4999 .0001 2.22 .4868 .0132 2.78 .4973 .0027 3.90 .49895 .00005	2.19	.4857	.0143	2.75	.4970	.0030	3.60	.4998	.0002
2.22 .4868 .0132 2.78 .4973 .0027 3.90 .49995 .00005	2.20						3.80	.4999	.0001
	2.22	.4868	.0132	2.78	.4973	.0027	3.90		.00005
2.23 .4871 .0129 2./9 .49/4 .0026 4.00 .49997 .00003	2.23	.4871	.0129	2.79	.4974	.0026	4.00	.49997	.00003

Reading the Z Table

Finding the proportion of observations between the mean and a score when ■ Z = 1.80

1.68 .4535 .0465 .0455 1.69 .4545 1.70 .4554 .0446 .0436 1.71 .4564 .4573 .0427 1.72 1.73 .4582 .0418 .4591 .0409 1.74 .0401 1.75 .4599 1.76 .4608 .0392 1.77 .4616 .0384 .0375 1.78 .4625 1.79 .4633 .0367 1.80 4641 .0359 .0351 1.81 .4649 .0344 1.82 .4656 .0336 1.83 .4664 .0329 1.84 .4671 .0322 1.85 .4678 1.86 .4686 .0314 1.87 .4693 0307 .0301 1.88 .4699 1.89 .4706 .0294 .4713 .0287 1.90 .0281 1.91 .4719 .0274 1.92 .4726

В

С

Α

z

Reading the Z Table

Finding the proportion .4535 1.68 .4545 1.69 of observations above a 1.70 .4554 1.71 .4564 .4573 1.72 score when .4582 1.73 1.74 .4591 ■ Z = 1.80 .4599 1.75 1.76 4608 1.77 4616 .4625 1.78 1.79 .4633 1.80 .4641 .4649 1.81 1.82 .4656 1.83 .4664 1.84 .4671 1.85 .4678 1.86 1.87 1.88 1.89 .4706 .4713 1.90 1.91 1.92 .4726

.0465 .0455 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .0375 .0367 .0359 .0351 .0344 .0336 .0329 .0322 .4686 .0314 .4693 .0307 .4699 .0301 .0294 .0287 .4719 .0281 .0274

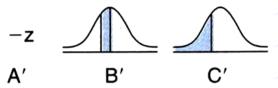
С

Α

z

В

Reading the Z Table	1.98	.4761	.0239
	1.99	.4767	.0233
	2.00	.4772	.0228
	2.01	.4778	.0222
	2.02	.4783	.0217
Finding the proportion	2.02 2.03 2.04 2.05 2.06 2.07	.4788 .4793 4798 .4803 .4808	.0217 .0212 .0207 .0202 .0197 .0192
of observations between a score and the mean when	2.08 2.09 2.10 2.11 2.12	.4812 .4817 .4821 .4826 .4830	.0188 .0183 .0179 .0174 .0170
■ Z = -2.10	2.13	.4834	.0166
	2.14	.4838	.0162
	2.15	.4842	.0158
	2.16	.4846	.0154
	2.17	.4850	.0150
	2.18	.4854	.0146
	2.19	.4857	.0143
	2.20	.4861	.0139
	2.21	.4864	.0136
	2.22	.4868	.0132



Reading the Z Table	1.98 1.99 2.00 2.01 2.02	.4761 .4767 .4772 .4778 .4783	.0239 .0233 .0228 .0222 .0217
 Finding the proportion of observations below a 	2.02 2.03 2.04 2.05 2.06 2.07	.4788 .4793 4798 .4803 .4808	.0212 .0207 .0202 .0197 .0192
score when Z = -2.10	2.08 2.09 2.10 2.11 2.12	.4812 .4817 .4821 .4826 .4830	.0188 .0183 .0179 .0174 .0170
	2.13 2.14 2.15 2.16 2.17	.4834 .4838 .4842 .4846 .4850	.0166 .0162 .0158 .0154 .0150
	2.18 2.19 2.20 2.21 2.22	.4854 .4857 .4861 .4864 .4868	.0146 .0143 .0139 .0136 .0132
	2.23 - z	.4871	.0129
	Α'	B'	C′

Z scores and the Normal Distribution

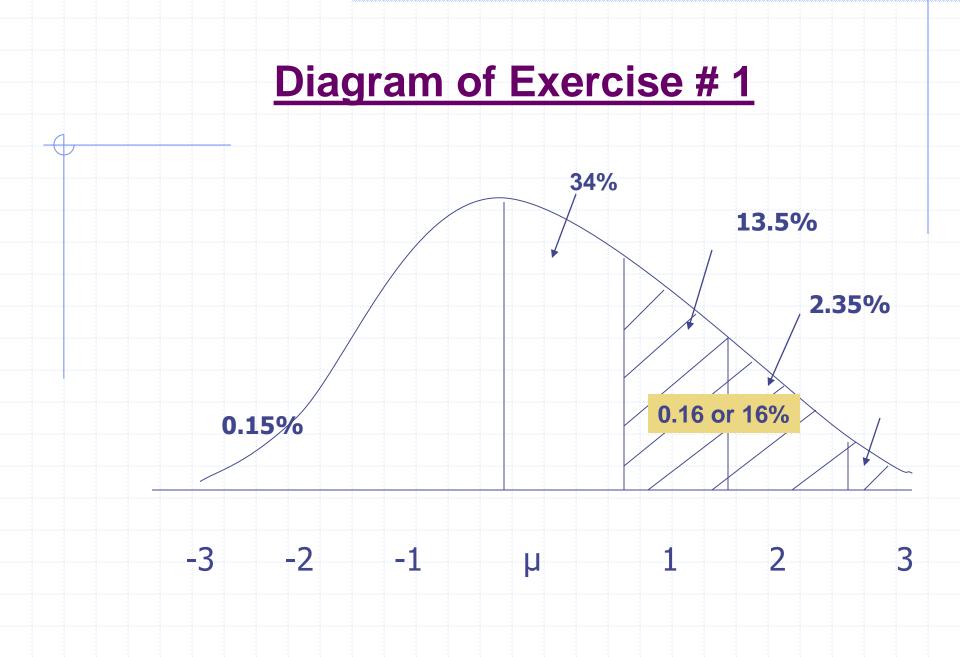
- Can answer a wide variety of questions about any normal distribution with a known mean and standard deviation.
- Will address how to solve two main types of normal curve problems:
 - Finding a proportion given a score.
 - Finding a score given a proportion.

Exercises

Assuming the normal heart rate (H.R) in normal healthy individuals is normally distributed with Mean = 70 and Standard Deviation =10 beats/min

Then:

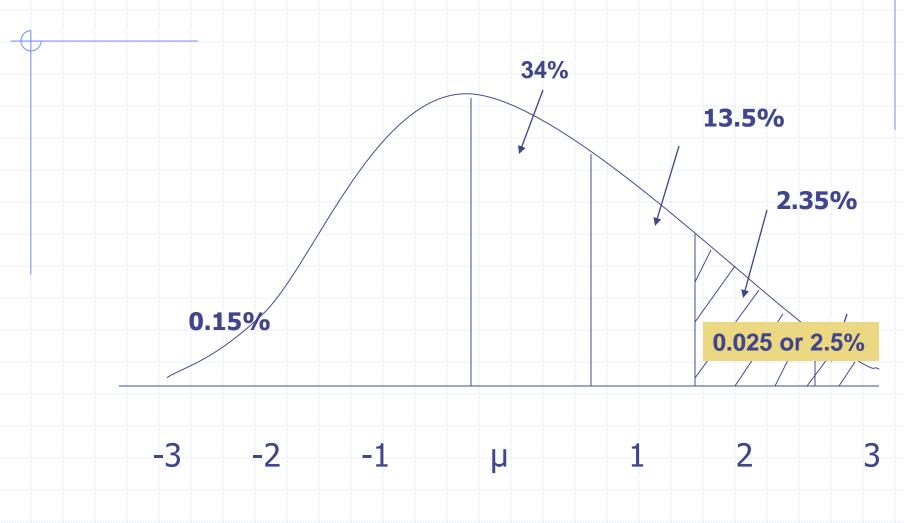
1) What area under the curve is above 80 beats/min?



Then:

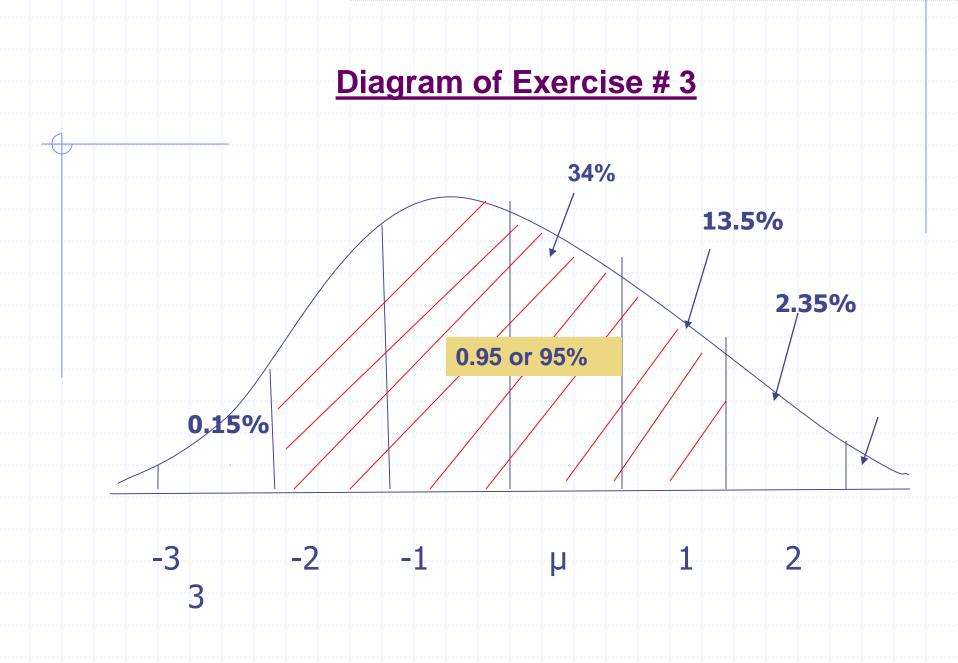
2) What area of the curve is above 90 beats/min?

Diagram of Exercise # 2



Then:

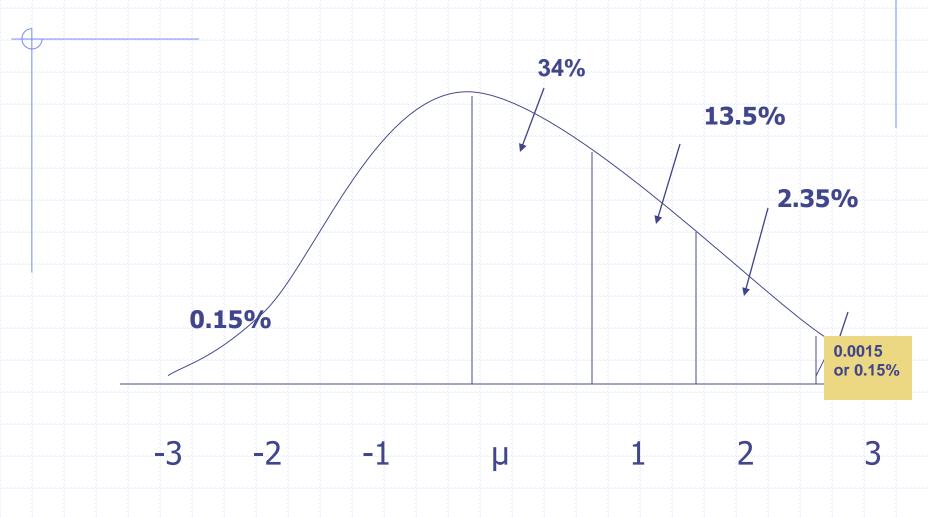
3) What area of the curve is between 50-90 beats/min?



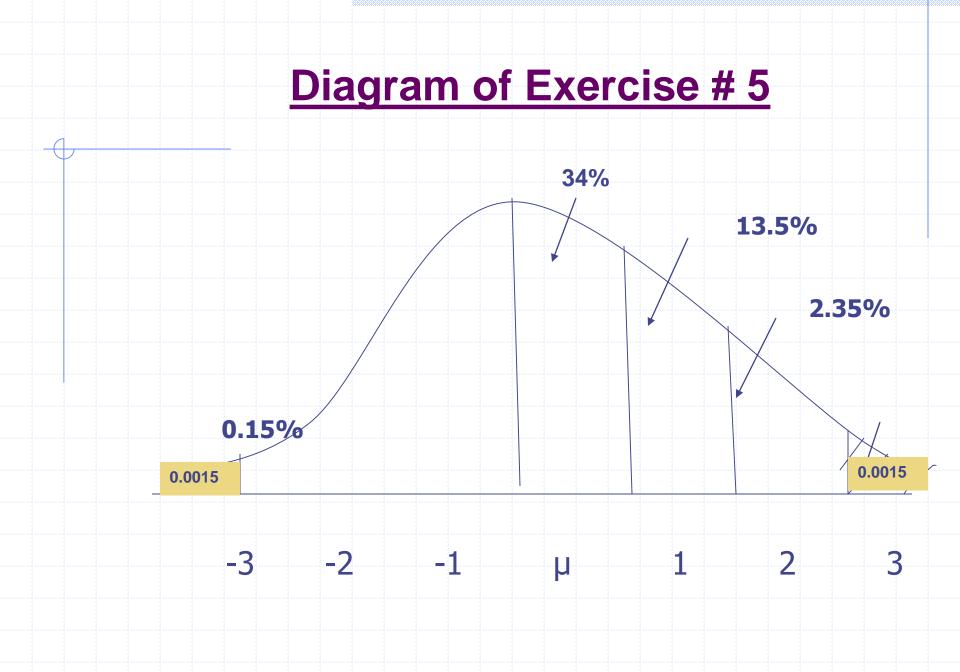
Then:

4) What area of the curve is above 100 beats/min?





5) What area of the curve is below 40 beats per min or above 100 beats per min?



Exercise:

Assuming the normal heart rate (H.R) in normal healthy individuals is normally distributed with Mean = 70 and Standard Deviation =10 beats/min

Then:

1) What area under the curve is above 80 beats/min? Ans: 0.16 (16%) 2) What area of the curve is above 90 beats/min? Ans: 0.025 (2.5%) 3) What area of the curve is between 50-90 beats/min? Ans: 0.95 (95%) 4) What area of the curve is above 100 beats/min? Ans: 0.0015 (0.15%)

5) What area of the curve is below 40 beats per min or above 100 beats per min?

Ans: 0.0015 for each tail or 0.3%

Problem:

Assume that among diabetics the fasting blood level of glucose is approximately normally distributed with a mean of 105mg per 100ml and an SD of 9 mg per 100 ml. What proportion of diabetics having fasting blood glucose levels between 90 and 125 mg per 100 ml ?

Let X be the random variable denoting the fasting blood glucose level. X has a normal distribution with mean = 105 and standard deviation = 9.

We have to compute $P(90 \le X \le 125)$. The table is available only for the probabilities of a standard normal distribution. Thus we have to convert X to i) a standard normal variable (Z), using the formula on page 5 of this module.

We require P (90 $\leq X \leq$ 125).

This can be written as

$$P\left[\frac{90-105}{9} \le \frac{X-105}{9} \le \frac{125-105}{9}\right] = P(-1.67 \le Z \le 2.22)$$

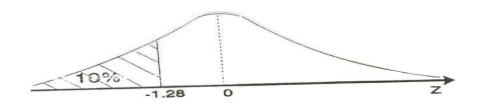
since $Z = \frac{X-105}{9}$
$$= P(Z \le 2.22) - P(Z < -1.67)$$

$$= 0.9868 - 0.0475$$

$$= 0.9393$$

8

Therefore 94% of diabetics have fasting blood glucose levels between 90 and 125.



From the table we know that -1.28 cuts off the lower 10 per cent of the standard normal curve. Now we have to find the corresponding X-value.

ii)



