Statistical tests for Quantitative variables (z-test, t-test & Correlation)

Objectives:

- Able to understand the factors to apply for the choice of statistical tests in analyzing the data .
- Able to apply appropriately Z-test, student's t-test & Correlation
- Able to interpret the findings of the analysis using these three tests.

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Resources:

• 436 Lecture Slides + Notes

Important – Notes - Extra

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Editing file

Feedback link

Summary

Choosing the appropriate Statistical test

Based on the three aspects of the data:

1. Type of variables
2. Number of groups being compared
3. Sample size

Z-test

Study variable: Qualitative (Categorical)
Outcome variable: Quantitative
Comparison : 1- sample mean with population mean
2- two sample means
Sample size : larger in each group(>30) & standard deviation is known

Comparing Sample mean with Population mean:

1. Make Assumptions and Meet Test Requirements
2. State the Null Hypothesis
3. State the Alternative Hypothesis
4. Select Sampling Distribution and Establish the Critical Region
5. Make a Decision and Interpret Results

Comparison of two sample means:

1. Determine the null and alternative hypotheses
2. Sampling distribution: Normal distribution (z-test)
3. Assumptions of test statistic (sample size > 30 in each group)
4. Collect and summarize data into a test statistic.
5. Determine the p-value
6. Make a decision

T-test

Study variable: Qualitative (Categorical)
Outcome variable: Quantitative
Comparison: 1-sample mean with population mean 2-two means (independent samples) 3-paired samples
Sample size : each group <30 (can be used even for (large sample size

Steps for test for single mean

1. Questioned to be answered
2. Null Hypothesis
3. Test statistics $t = \frac{\dot{x} - u}{s/\sqrt{n}}$
4. Comparison with theoretical value
5. Inference

Z- value & t-Value

"Z and t" are the measures of:
How difficult is it to believe the null hypothesis?

1. High z & t values:
Difficult to believe the null hypothesis -accept that there is a real difference.
2. Low z & t values:
Easy to believe the null hypothesis - have not proved any difference.

Correlation Coefficient:

a correlation coefficient (r) provides a quantitative way to express the degree of linear relationship between two variables.
1- Range: r is always between -1 and 1
2- Sign of correlation indicates direction: - high with high and low with low -> positive,
- high with low and low with high -> negative, - no consistent pattern -> near zero
3- Magnitude (absolute value) indicates strength (-.9 is just as strong as .9)

Correlation Coefficient: Limitations

1. Correlation coefficient is appropriate measure of relation only when relationship is linear
2. Correlation coefficient is appropriate measure of relation when equal ranges of scores in the sample and in the population.
3. Correlation doesn't imply causality

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Choosing the appropriate Statistical test , (very important).

Based on the three aspects of the data: (these three are very important)

1. Type of variables {Qualitative (Categorical), Quantitative}
2. Number of groups being compared (how much group you are willing to compare)
3. Sample size (in Z test its high while in t test its low)

Statistical Tests:

A statistical test provides a mechanism for making quantitative decisions about a process or processes. The intent is to determine whether there is enough evidence to "reject" a hypothesis about the process.

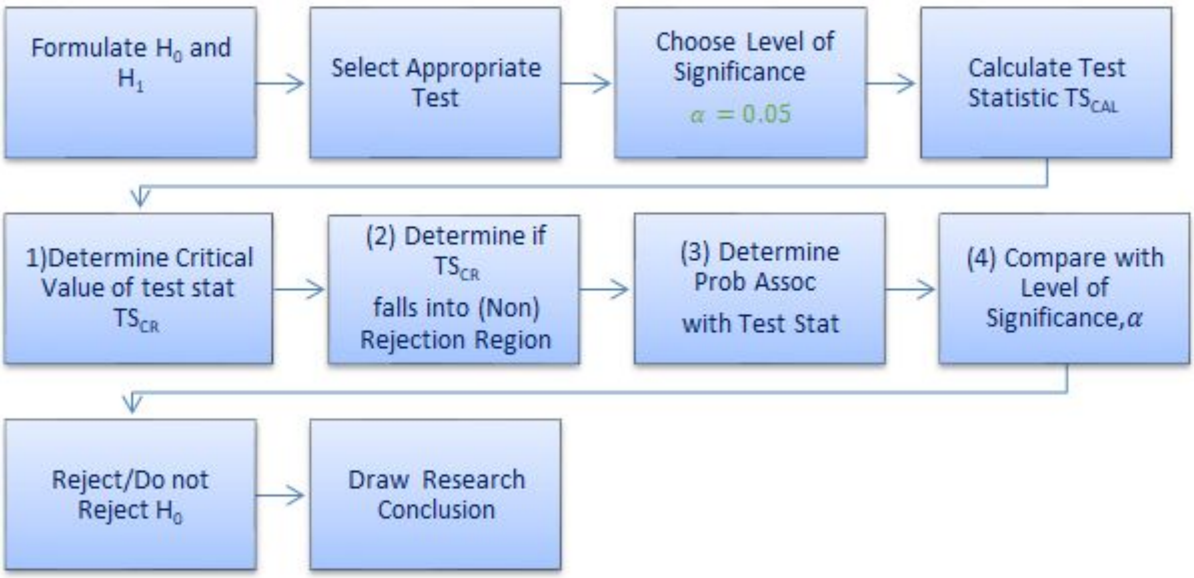
❖ Z-test: (also known as normal distribution test)

Study variable:	Qualitative (Categorical)
Outcome variable:	Quantitative
Comparison:	i. sample mean with population mean ii. two sample means
Sample size:	larger in each group(>30) & standard deviation is known

The test statistic is assumed to have a normal distribution, also parameters such as standard deviation should be known in order for an accurate z-test to be performed.

Steps for Hypothesis Testing

Hypothesis testing is just a way for you to figure out if results from a test are valid or repeatable. Assume that someone said they found a new drug that cures some disease, you would want to be sure it was probably true or not. How? A hypothesis test will tell you if it's probably true, or probably not true.



I. Comparing Sample mean with Population mean :

Example

- The education department at a university has been accused of "grade inflation" (grade increasing) in medical students with higher GPAs than students in general.
- GPAs of all medical students should be compared with the GPAs of all other (non-medical) students.
  - There are 1000s of medical students, far too many to interview.
  - How can this be investigated without interviewing all medical students ?

What we know:

- The average GPA for all other students is 2.70. This value is a parameter.
  $\mu = 2.70$

- To the right is the statistical information for a random sample of medical students:

$\bar{x}$ =	3.00
s =	0.70
n =	117

Questions to ask:

- **Is there a difference between the parameter (2.70) and the statistic (3.00)?**
- **Could the observed difference have been caused by random chance?**
- **Is the difference real (significant)?**

1. The sample mean (3.00) is the same as the pop. mean (2.70). **this is null hypothesis**
   - The difference is trivial and caused by random chance.
2. The difference is real (significant). **this is the assumption.**
   - Medical students are different from all other students.

➢ Step 1: Make Assumptions and Meet Test Requirements

- Random sampling
  - Hypothesis testing assumes samples were selected using random sampling.
  - In this case, the sample of 117 cases was randomly selected from all medical students.

- Level of Measurement is interval-Ratio.

GPA is I-R , so the mean is an appropriate statistic.

- Sampling Distribution is normal in shape

This is a "large" sample ($n \geq 100$).

➢ Step 2\ i. State the Null Hypothesis

- H_0: μ=2.7 (in other words, $H_0: \bar{x} = \mu$)

- We can state H_0 : No difference between the sample mean and the population parameter
- (In other words, the sample mean of 3.0 really the same as the population mean of 2.7 – the difference is not real but is due to chance.)
- The sample of 117 comes from a population that has a GPA of 2.7.
- The difference between 2.7 and 3.0 is trivial and caused by random chance.

➢ Step 2 \ ii. State the Alternative Hypothesis

- H_0: μ ≠2.7 (or, $H_0: \bar{x} \neq \mu$)

- Or H_1 : There is a difference between the sample mean and the population parameter
- The sample of 117 comes a population that does not have a GPA of 2.7. In reality, it comes from a different population.
- The difference between 2.7 and 3.0 reflects an actual difference between medical students and other students.
- Note that we are testing whether the population the sample comes from is from a different population or is the same as the general student population.

➢ Step 3 Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution= Z

- Alpha (α) = .05
- α is the indicator of "rare" events.
- Any difference with a probability less than α is rare and will cause us to reject the H0.

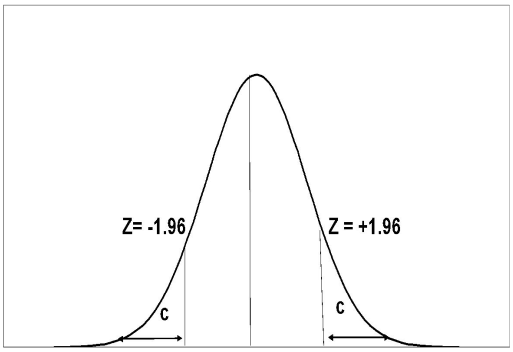
- Critical Region begins at Z= ± 1.96

- This is the critical Z score associated with α = .05, two-tailed test.
- If the obtained Z score falls in the Critical Region, or "the region of rejection," then we would reject the H0.

When the Population σ is not known, use the following formula: $z = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

Test the Hypotheses: $z = \dfrac{3.0 - 2.7}{0.7/\sqrt{117-1}} = 4.62$ this is your calculated z-value, where does it fall? look at the normal curve.

Substituting the values into the formula, we calculate a Z score of 4.62.



Two-tailed Hypothesis Test

- When α = .05, then .025 of the area is distributed on either side of the curve in area (C)
- The .95 in the middle section represents no significant difference between the population and the sample mean.
- The cut-off between the middle section and +/- .025 is represented by a Z-value of +/- 1.96.

➢ Step 5 Make a Decision and Interpret Results

- The obtained Z score fell in the Critical Region, so we reject the H_0.

- If the H_0 were true, a sample outcome of 3.00 would be unlikely.
- Therefore, the H_0 is false and must be rejected.

- Medical students have a GPA that is significantly different from the non-medical students (Z = 4.62, $p < 0.05$).

Summary:

- The GPA of medical students is significantly different from the GPA of non-medical students.
- In hypothesis testing, we try to identify statistically significant differences that did not occur by random chance.
- In this example, the difference between the parameter 2.70 and the statistic 3.00 was large and unlikely ($p < .05$) to have occurred by random chance.

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II. Comparison of two sample means

Example : Weight Loss for Diet vs Exercise

Did dieters lose more fat than the exercisers?

	Diet Only	Exercise Only
Sample mean	5.9 kg	4.1 kg
Sample standard deviation	4.1 kg	3.7 kg
Sample size (n)	42	47
Standard error	$SEM_1 = 4.1/\sqrt{42} = 0.633$	$SEM_2 = 3.7/\sqrt{47} = 0.540$

measure of variability (pooled standard error) = $\sqrt{[(0.633)^2 + (0.540)^2]} = 0.83$

The simple mean difference = sample mean of the diet – sample mean of the exercise → 1.8 kg

we have to prove whether 1.8 is due to chance, or because of the intervention

- Step 1. Determine the null and alternative hypotheses.

  Null hypothesis: No difference in average fat lost in population for two methods. Population mean difference is zero.

  Alternative hypothesis: There is a difference in average fat lost in population for two methods. population mean difference is not zero.

- Step 2. Sampling distribution: Normal distribution (z-test)
- Step 3. Assumptions of test statistic (sample size > 30 in each group)
- Step 4. Collect and summarize data into a test statistic.

  The sample mean difference = 5.9 – 4.1 = 1.8 kg
  and the standard error of the difference is 0.83.

  So the test statistic: $z = \frac{1.8-0}{0.83} = 2.17$

- Step 5. Determine the p-value.

  Recall the alternative hypothesis was two-sided.
  p-value = 2 × [proportion of bell-shaped curve above 2.17]
  Z-test table = > proportion is about 2 × 0.015 = 0.03.

- Step 6. Make a decision.

  The p-value of 0.03 is less than or equal to 0.05, so …

  - If really no difference between dieting and exercise as fat loss methods, would see such an extreme result only 3% of the time, or 3 times out of 100. 0.03 means if we repeat the study 100 times, this result will come 3 times, very rare.
  - Prefer to believe truth does not lie with null hypothesis. We conclude that there is a statistically significant difference between average fat loss for the two methods.

❖ Student's t-test: (compares two means and tells you if they are any different from each other or not. If there is a deference, it will tell you the this difference significant or not)

Study variable:	Qualitative (Categorical)
Outcome variable:	Quantitative
Comparison:	i. sample mean with population mean ii. two means (independent samples) iii. paired samples
Sample size:	each group <30 (can be used even for large sample size) Memorize that the sample size here is small while in z test was high

Tests types:

1. **Test for single mean**

Whether the sample mean is equal to the predefined population mean?

2. Test for difference in means

Whether the CD4 level of patients taking treatment A is equal to CD4 level of patients taking treatment B ?

3. Test for paired observation

Whether the treatment conferred any significant benefit ?

1.Steps for test for <u>single mean</u>

1. Questioned to be answered

   Is the Mean weight of the sample of 20 rats is 24 mg?

   N=20, $\dot{x}$ =21.0 mg, sd=5.91 , μ =24.0 mg

2. Null Hypothesis

   The mean weight of rats is 24 mg. That is, The sample mean is equal to population mean.

3. Test statistics $t = \dfrac{\dot{x} - u}{s/\sqrt{n}}$ • $t_{(n-1)}$df This is degree of freedom, which is here: sample size - 1 so, 20-1 = 19 df

4. Comparison with theoretical value

   if tab $t_{(n-1)}$ < cal $t_{(n-1)}$ reject Ho,

   if tab $t_{(n-1)}$ > cal $t_{(n-1)}$ accept Ho,

5. Inference

t –test for single mean

- Test statistics

  n=20, $\dot{x}$ =21.0 mg, sd=5.91 , μ =24.0 mg

$$t = \frac{|21.0 - 24|}{5.91/\sqrt{20}} = 2.30$$

2.30 is the calculated t value from the data.

t_α= $t_{.05,\ 19}$= 2.093

Accept H0 if t < 2.093

Reject H0 if t >= 2.093

Inference : We reject Ho, and conclude that the data is not providing enough evidence, that the sample is taken from the population with mean weight of 24 gm

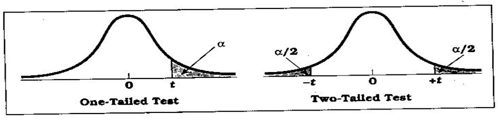

Table D.6 **Percentage Points of the *t* Distribution** (Source: The entries in this table were computed by the author.)

	Level of Significance for One-Tailed Test								
	.25	.20	.15	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

what is table value? like z-table we have student t distribution also. you have two rows with different level of significance for one\two tailed test.
so for 19 df and 0.05 (two tailed test) the value is 2.093. this is what you use to compare the calculated t value.
2.3 is higher than 2.093, so you reject null hypothesis.

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2. t-test for <u>difference in means</u>

Given below are the 24 hrs total energy expenditure (MJ/day) in groups of lean and obese women. Examine whether the obese women's mean energy expenditure is significantly higher ?.

Lean

7.5	7.0	6.1
7.6	5.5	7.5
8.1	8.1	7.9
10.2	8.4	8.1
		10.9

obese

9.2	9.2	8.8
10.0	9.7	9.7
12.8	11.8	11.5

Null Hypothesis

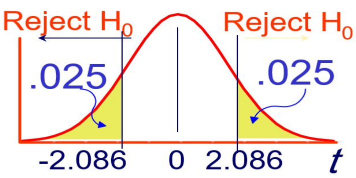
Obese women's mean energy expenditure is equal to the lean women's energy expenditure.

Solution

H_0: $\mu_1 - \mu_2 = 0$ $(\mu_1 = \mu_2)$
H_1: $\mu_1 - \mu_2 \neq 0$ $(\mu_1 \neq \mu_2)$
$\alpha = 0.05$
df = $n_1 + n_2 - 2$ = df = 13 + 9 - 2 = 20 (2 samples here)
difference of means: 10.3 - 8.1 = 2.2 (we have to prove is this due to chance or not)

Data Summary		
	lean	obese
N sample size	13	9
mean	8.10	10.30
S standard deviation	1.38	1.25

Critical Value(s):



Calculating the Test Statistic:

- Compute the Test Statistic:

Hypothesized Difference (usually zero when testing for equal means)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

You don't have to memorize these formula

Developing the Pooled-Variance t Test

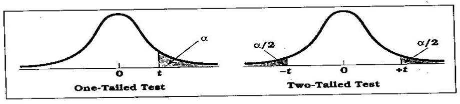
- Calculate the Pooled Sample Variances as an Estimate of the Common Populations Variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

s_p^2 = Pooled-Variance
s_1^2 = Variance of Sample 1
n_1 = Size of Sample 1
s_2^2 = Variance of sample 2
n_2 = Size of Sample 2

First, estimate the common variance as a weighted average of the two sample variances using the degrees of freedom as weights

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1-1)+(n_2-1)} = \frac{(13-1)1.38^2+(9-1)1.25^2}{(13-1)+(9-1)} = 1.765$$



Calculating the Test Statistic:

$$t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_p^2\cdot\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{|8.1-10.3|-0}{1.76\sqrt{\frac{1}{13}+\frac{1}{9}}} = 3.82$$

tab t 9+13-2 =20 dff = t 0.05,20 =2.086

Inference : The cal t (3.82) is higher than tab t at 0.05, 20. ie 2.086 . This implies that there is a evidence that the mean energy expenditure in obese group is significantly (p<0.05) higher than that of lean group

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∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Example

Suppose we want to test the effectiveness of a program designed to increase scores on the quantitative section of the Graduate Record Exam (GRE). We test the program on a group of 8 (small n) students. Prior to entering the program, each student takes a practice quantitative GRE; after completing the program, each student takes another practice exam. Based on their performance, was the program effective? 2scores for one student (before and after).

Each subject contributes 2 scores: repeated measures design

Student	Before Program	After Program
1	520	555
2	490	510
3	600	585
4	620	645
5	580	630
6	560	550
7	610	645
8	480	520

Can represent each student with a single score: the difference (D) between the

Student	D
1	35
2	20
3	-15
4	25
5	50
6	-10
7	35
8	40

- **Approach:** test the effectiveness of program by testing significance of D
- Null hypothesis: There is no difference in the scores of before and after program
- Alternative hypothesis: program is effective → scores after program will be higher than scores before program → average D will be greater than zero

one sided because we know that the program is going to be effective, interpretation will go in the positive direction

$$H_0: \mu_D = 0$$
$$H_1: \mu_D > 0$$

So, need to know $\sum D$ and $\sum D^2$:

Student	Before Program	After Program	D	D^2
1	520	555	35	1225
2	490	510	20	400
3	600	585	-15	225
4	620	645	25	625
5	580	630	50	2500
6	560	550	-10	100
7	610	645	35	1225
8	480	520	40	1600
			$\sum D = 180$	$\sum D^2 = 7900$

Recall that for single samples: $t_{obt} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{score \; - mean}{standard \; error}$

for related samples: $t_{obt} = \frac{\bar{D} - \mu_D}{s_{\bar{D}}}$

Where: $s_{\bar{D}} = \frac{s_D}{\sqrt{N}}$ and $S_D = \sqrt{\frac{\sum D^2 - \frac{(\Sigma D)^2}{N}}{N-1}}$

Mean of D: $\bar{D} = \frac{\Sigma D}{N} = \frac{180}{8} = 22.5$

Standard deviation of D: $S_D = \sqrt{\frac{\sum D^2 - \frac{(\Sigma D)^2}{N}}{N-1}} = \sqrt{\frac{7900 - \frac{(180)^2}{8}}{8-1}} = 23.45$

Standard error: $s_{\bar{D}} = \frac{s_D}{\sqrt{N}} = \frac{23.45}{\sqrt{8}} = 8.2908$

$t_{obt} = \frac{\bar{D} - \mu_D}{s_{\bar{D}}}$ Under H_0, μ_0 =0, so: $t_{obt} = \frac{\bar{D}}{s_{\bar{D}}} = \frac{22.5}{8.2908} = 2.714$

From table B.2: for α = 0.05, one tailed, with df=7

$t_{critical} = 1.895$

$2.714 > 1.895 \rightarrow reject \; H_0$

The program is effective.

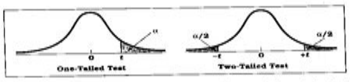


Table D.6 **Percentage Points of the t Distribution** (Source: The entries in this table were computed by the author.)

	Level of Significance for One-Tailed Test								
	.25	.20	.15	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Z- value & t-Value

"Z and t" are the measures of:

How difficult is it to believe the null hypothesis?

1. High z & t values:
   Difficult to believe the null hypothesis -accept that there is a real difference.
2. Low z & t values:
   Easy to believe the null hypothesis - have not proved any difference.

- I will give you a clue, whatever you calculate from the data(Z OR T), when these values are higher, your p value will be smaller.

	Z test	T test
	How the means of two sets of data differ from one another when **variance are given.**	How the means of two sets of data differ from one another when **variance are not given.**
Sample size	Large	Low

❖ Karl Pearson Correlation Coefficient

A number called the correlation measures both the direction and strength of the linear relationship between two related sets of quantitative variables.

This definition is very important. If we have any correlation pictures we have to explain these terms (it will be more clear in pics)

1- direction: it means is it positive or negative. How we determine that? By the graph directions or by the mines before the number.

2- strength: also known as magnitude which mean is it weak, moderate, or strong. How can we determine that? If all the variables are increasing at the same time (one perfect line) its strong even if it's positive or negative, if there is some differences its moderate and if it completely different it is weak

3- linear: as one increases the other will increase also, and vise versa

Working with two variables (parameter)

As Age ↑ — BP ↑
As Height ↑ — Weight ↑
As Age ↑ — Cholesterol ↑
As duration of HIV ↑ — CD4 CD8 ↓

The first three go at the same directions. When the age increased the BP will increased and the cholesterol will increase also, the height increased the weight will increase. Same direction = positive direction

While the last one go in the opposite direction. Opposite direction = negative directions

Types of correlation

Negative Correlation	Positive correlation
• Variables move in opposite direction	• Variables move in the same direction
:Examples ▪ Duration of HIV/AIDS and CD4 CD8 ▪ Price and Demand ▪ Sales and advertisement expenditure	:Examples ▪ Height and Weight ▪ Age and BP

Measurement of correlation:

1. Scatter Diagram
2. Karl Pearson's coefficient of Correlation

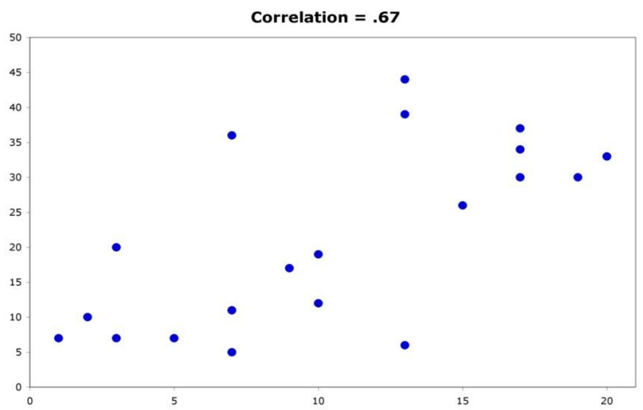
Graphical Display of Relationship

- Scatter diagram
- Using the axes
  - X-axis horizontally
  - Y-axis vertically
  - Both axes meet: origin of graph: 0/0
  - Both axes can have different units of measurement
  - Numbers on graph are (x,y)

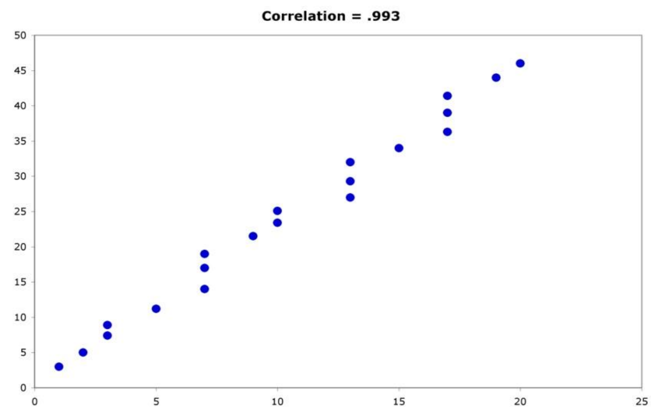
Guess the Correlations:

.67 .993 .003 -.975

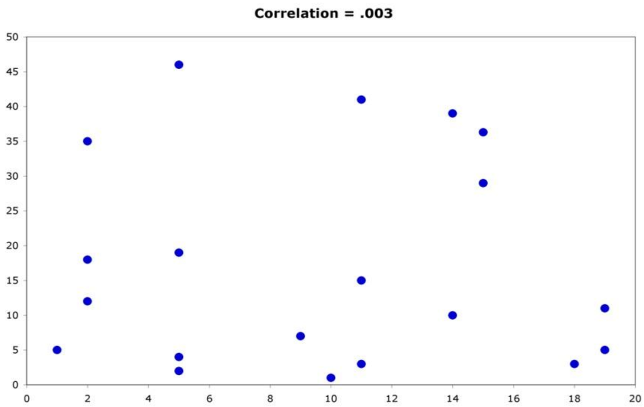
If we have any pictures answer these three
What is direction? Positive or negative? How ? In the previous slide
Degree of strength



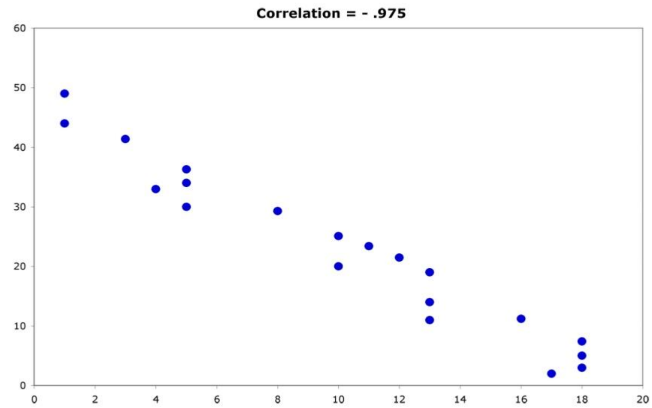
- What is direction? Positive. Why ? Because 0.67 is positive number and its go from down to up.
- What is degree of strength? Moderate
So we said ⟶ its positive moderate correlation



- What is direction? Positive. Why ? Because 0.993 is positive number and its go from down to up. والعدد قريب من الواحد
- What is degree of strength? Strong
- So we said ⟶ its positive perfect or strong correlation



- What is direction? Positive. Why ? Because 0.67.
- What is degree of strength? Weak
- So we said ⟶ its positive very weak correlation or there is no any correlation



- What is direction? negative. Why ? Because - 0.975 is positive number and its go from up to down.
- What is degree of strength? Strong
- So we said ⟶ its negative perfect or strong correlation

The pearson r: $r = \frac{\Sigma XY - \frac{(\Sigma x)(\sum Y)}{N}}{\sqrt{\left[\sum X^2 - \frac{(\Sigma X)^2}{N}\right]\left[\sum Y^2 - \frac{(\Sigma Y)^2}{N}\right]}}$

We Need:

- Sum of the Xs ΣX
- Sum of the Ys ΣY
- Sum of the Xs squared $(\Sigma X)^2$
- Sum of the Ys squared $(\Sigma Y)^2$
- Sum of the squared Xs ΣX^2
- Sum of the squared Ys ΣY^2
- Sum of Xs times the Ys ΣXY
- Number of Subjects (N)

Example:

A sample of 6 children was selected, data about their age in years and weight in kilograms was recorded as shown in the following table . Find the correlation between age and weight.

Weight ((Kg	Age ((years	serial No
12	7	1
8	6	2
12	8	3
10	5	4
11	6	5
13	9	6

Y^2	X^2	xy	Weight ((Kg (y)	Age ((years (x)	Serial .n
144	**49**	**84**	**12**	**7**	**1**
64	**36**	**48**	**8**	**6**	**2**
144	**64**	**96**	**12**	**8**	**3**
100	**25**	**50**	**10**	**5**	**4**
121	**36**	**66**	**11**	**6**	**5**
169	**81**	**117**	**13**	**9**	**6**
=y^2∑ 742	**=x^2∑ 291**	**xy= 461∑**	**=y∑ 66**	**=x∑ 41**	**Total**

$$r = \frac{461 - \frac{(41)(66)}{6}}{\sqrt{\left[291 - \frac{(41)^2}{6}\right]\left[742 - \frac{(66)^2}{6}\right]}}$$

r = 0.759

strong direct correlation or strong positive relationship.

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Example: Relationship between Anxiety and Test Scores

Anxiety (X)	Test score (Y)	X^2	Y^2	XY
10	2	100	4	20
8	3	64	9	24
2	9	4	81	18
1	7	1	49	7
5	6	25	36	30
6	5	36	25	30
$\sum X = 32$	$\sum Y = 32$	$\sum X^2 = 230$	$\sum Y^2 = 204$	$\sum XY = 129$

Calculating Correlation Coefficient

$$r = \frac{(6)(129)-(32)(32)}{\sqrt{(6(230)-32^2)(6(204)-32^2)}} = \frac{774-1024}{\sqrt{(356)(200)}} = -0.94$$

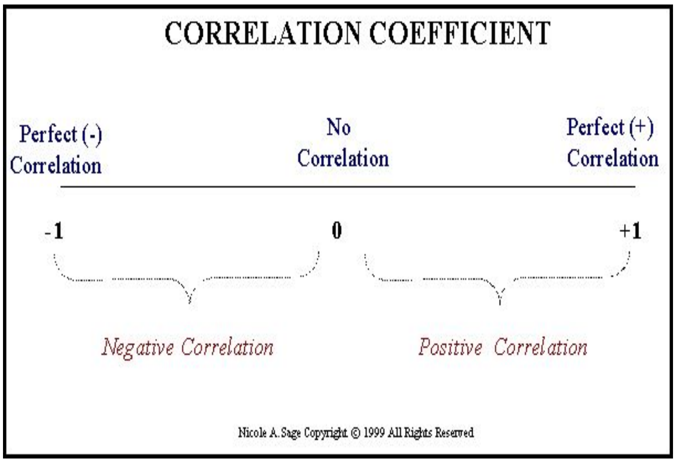
r = - 0.94

Indirect strong correlation or negative strong relationship

Correlation Coefficient

a correlation coefficient (r) provides a quantitative way to express the degree of linear relationship between two variables.

- Range: r is always between -1 and 1
- Sign of correlation indicates direction:
  - high with high and low with low -> positive
  - high with low and low with high -> negative
  - no consistent pattern -> near zero
- Magnitude (absolute value) indicates strength (-.9 is just as strong as .9)
  - .10 to .40 weak
  - .40 to .80 moderate
  - .80 to .99 high
  - 1.00 perfect



About "r"

- r is not dependent on the units in the problem
- r ignores the distinction between explanatory and response variables
- r is not designed to measure the strength of relationships that are not approximately straight line
- r can be strongly influenced by outliers. extreme values. so you must look for the extreme values and take them out.

Correlation Coefficient: Limitations

1. Correlation coefficient is appropriate measure of relation <u>only</u> when relationship is linear
2. Correlation coefficient is appropriate measure of relation when equal ranges of scores in the sample and in the population. you can't relate BP of U.S data to age of saudi data. must be within the same sample,population
3. Correlation doesn't imply causality
   - Using U.S. cities a cases, there is a strong positive correlation between the number of churches and the incidence of violent crime
   - Does this mean churches cause violent crime, or violent crime causes more churches to be built? no
   - More likely, both related to population of city (3d variable -- lurking or confounding variable)

> - Ice-cream sales are strongly correlated with crime rates. Therefore, ice-cream causes crime.
>
> it makes no sense, people are in crowded area, because of that there is a lot of ice-cream stores. and because there is people, crimes are happening

Without proper interpretation, causation should not be assumed, or even implied.

In conclusion !

- Z-test will be used for both categorical(qualitative) and quantitative outcome variables. remember this!
- Student's t-test will be used for only quantitative outcome variables. remember this!
- Correlation will be used to quantify the linear relationship between two quantitative variables