Statistical tests to observe the statistical significance of qualitative variables

(Z-test, Chi-square, Fisher's exact & Mac Nemar's Chi-square)

Objectives:

- Able to understand the factors to apply for the choice of statistical tests in analyzing the data .
- Able to apply appropriately Z-test, Chi-square test, Fisher's exact test & Macnemar's Chi-square test.
- Able to interpret the findings of the analysis using these four tests.

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في الاختبار يجيك سيناريو ويبغى منك تحدد اي نوع اختبار تستخدم اهم شيء تعرفون كل اختبار ومتى نستخدمه واذا فيه مميزات مثلا كم لازم تكون العينة،في الاختبار مافيه مسائل حسابية

Resources:

- 436 Lecture Slides + Notes

Important – Notes

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Editing file

Feedback link

الحمد لله الذي بنعمته تتم الصالحات تم الانتهاء من آخر محاضرة لمادة البحث ،من لا يشكر الناس لا يشكر الله نخص بالشكر الجزيل:

القادة الأكاديميين : باسل المفلح ،مها الغامدي ، عبدالعزيز العنقري

وللأعضاء الذين هم أساس هذا العمل:

- ابتسام المطيري
- بدريه الصباغ
- جواهر ابانمي
- دعاء وليد
- ذكريات عمر
- غيداء السعيد
- لمى الفوزان
- ليلى مذكور
- منيرة العيوني
- منيرة الضفيان
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- وئام بابعير
- وجدان الزيد
- حاتم النداح
- خالد الحسينان
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- محمد العايد
- محمد غندور
- مزيد العتيبي
- عبد المحسن الغنام

فإن أصبنا فمن الله، وإن أخطأنا فمن أنفسنا والشيطان

لا تنسونا من خالص دعواتكم

مع تمنياتنا لكم بالتوفيق ، قادة الفريق : محمد اليوسف ،روان الوادعي

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lecture outline:

Types of Qualitative/ Categorical Data

- Nominal Category
- Ordinal Categories

Statistical Tests

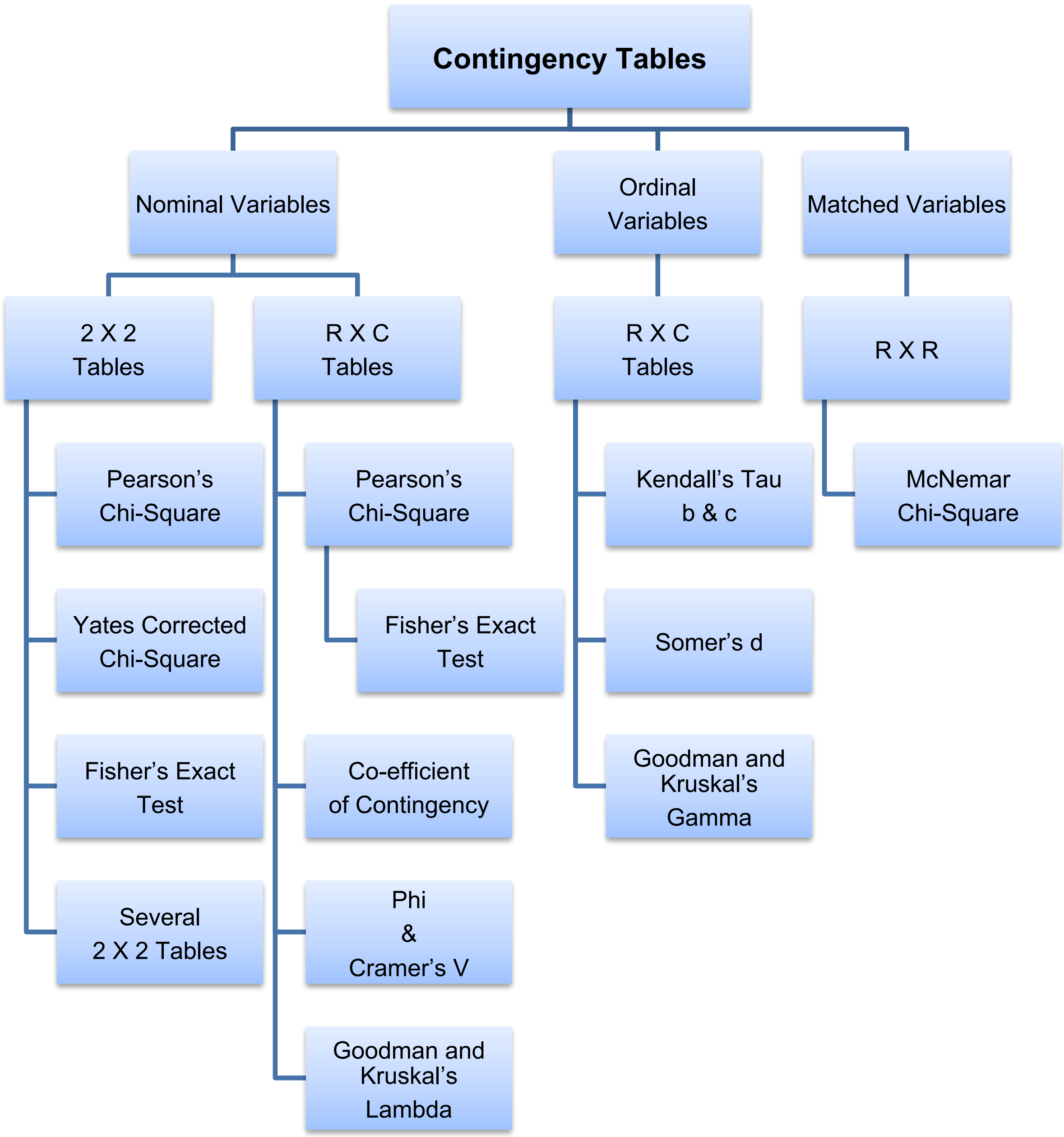
- **Z-test**
  - **Study variable:** Qualitative (Categorical)
  - **Outcome variable:** Qualitative (Categorical)
  - **Comparison:** 1- sample proportion with population proportion; 2- two sample proportions
  - **Sample size:** larger in each group (>30)
- **Chi-square test**
  - **Study variable:** Qualitative (Categorical)
  - **Outcome variable:** Qualitative (Categorical)
  - **Comparison:** two or more proportions
  - **Sample size:** 30<
  - **Expected frequency:** 5<
- **Fisher's exact test**
  - $= \frac{R_1!R_2!C_1!C_2!}{n!a!b!c!d!}$
  - **Study variable:** Qualitative (Categorical)
  - **Outcome variable:** Qualitative (Categorical)
  - **Comparison:** two proportions
  - **Sample size:** 30 >
- **Macnemar's test**
  - **Study variable:** Qualitative (Categorical)
  - **Outcome variable:** Qualitative (Categorical)
  - **Comparison:** two proportions
  - **Sample size:** any

Types of Qualitative/ Categorical Data

- Nominal Category
- Ordinal Categories

Types of Analysis for Categorical Data

- Descriptive (frequencies, percentages, Rate and Ratio)
- Analytical Test of Significance (p-value) and CI.



Choosing the appropriate Statistical test. Very important

Based on the three aspects of the data

- Types of variables
- Number of groups being compared
- Sample size

Everytime you are asked about the appropriate test ask these questions type of the variables Quantitative vs qualitative (Categorical)? Number of groups? sample size more than or less than 30?

Statistical Tests:

❖ Z-test:

Study variable:	Qualitative (Categorical)
Outcome variable:	Qualitative(Categorical)
Comparison:	i. sample proportion with population proportion; ii. two sample proportions
Sample size:	larger in each group(>30)

whenever you have the quantitative data you are calculating mean values. whenever you have qualitative data you are calculating proportions. this is a clue to understand the scenarios.

Test for sample proportion with population proportion. In the exam you will be given scenario exactly like this :,)

Problem

In an otological examination of school children, out of 146 children examined 21 were found to have some type of otological abnormalities. Does it confirm with the statement that 20% of the school children have otological abnormalities?

a. Question to be answered:

Is the sample taken from a population of children with 20% otological abnormality?

20% came from another study, the aim is to test it. The 20% of 146 is 14.4…. It's not equal to 21 mentioned above. So test the difference, is it due to by chance? or this school has less prevalence of abnormality. The result of z score test is 1.69, the next step is to look in z table and find the no, in raw=1.6 & column=0.09, the number is 0.05. 0.05 means 0.96 above 1(the average), so 1 & 0.96=1.96. So u look at the test statistic number, if it's less than 1.96 then accept the null hypothesis. If it's above then reject the null hypothesis.

b. Null hypothesis :

The sample has come from a population with 20% otological abnormal children

c. Test statistics $z = \frac{|p-p|}{\sqrt{\frac{pq}{n}}} = \frac{|14.4-20|}{\sqrt{\frac{14.4*85.6}{146}}} = 1.69$

P – Population Prop.
p- sample prop.
n- number of samples

21/146=0.143*100=14.4

100-14.4=85.6

d. Comparison with theoretical value

Z ~ N (0,1); $z_{0.05}$= 1.96

The prob. of observing a value equal to or greater than 1.69 by chance is more than 5%. We therefore do not reject the Null Hypothesis

e. Inference

There is a evidence to show that the sample is taken from a population of children with 20% abnormalities

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Comparison of two sample proportions

Problem

In a community survey, among 246 town school children, 36 were found with conductive hearing loss and among 349 village school children 61 were found with conductive hearing loss. Does this data, present any evidence that conductive hearing loss is as common among town children as among village children?

a. Question to be answered:

Is there any difference in the proportion of hearing loss between children living in town and village?

Given data	sample 1	sample 2	
size	246	342	
hearing loss	36	61	
% hearing loss	14.6 %	17.5%	36/246=0.146*100=14.6%

b. Null Hypothesis

There is no difference between the proportions of conductive hearing loss cases among the town children and among the village children

c. Test statistics

$$Z = \frac{|p1-p2|}{\sqrt{\frac{p1q1}{n1}+\frac{p2q2}{n2}}} = \frac{|14.6-17.5|}{\sqrt{\frac{14.6*85.4}{246}+\frac{17.5*82.5}{342}}}$$

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q= 1- p

p1, p2 are sample proportions, n1,n2 are subjects in sample 1 & 2

d. Comparison with theoretical value

Z ~ N (0,1); $z_{0.05}$ = 1.96

The prob. of observing a value equal to or greater than 1.81 by chance is more than 5%. We therefore do not reject the Null Hypothesis

e. Inference

There is no evidence to show that the two sample proportions are statistically significantly different. That is, there is no statistically significant difference in the proportion of hearing loss between village and town, school children.

❖ Chi-square test:

study variable:	(Qualitative (Categorical
Outcome variable:	(Qualitative(Categorical
Comparison:	two or more proportions
Sample size:	X>30
Expected frequency:	X> 5

if you have 2 proportion you can use this test or z test but more than 2 proportion you use this

The data must satisfy these two conditions. (The last two).

Purpose

To find out whether the association between two categorical variables are statistically significant

Null Hypothesis

There is no association between two variables

$$x^2 = \sum \left[\frac{(o - e)^2}{e} \right]$$

Figure for Each Cell

1. The summation is over all cells of the contingency table consisting of r rows and c columns
2. O is the observed frequency
3. $\hat{E}$ is the expected frequency

$$\hat{E} = \frac{\text{(total of row in which the cell lies)} \bullet \text{(total of column in which the cell lies)}}{\text{(total of all cells)}}$$

reject H_o if $\chi^2 > \chi^2_{.\alpha,df}$

where df = $(r\text{-}1)(c\text{-}1)$

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

4. The degrees of freedom are df = (r-1)(c-1)

Requirements

- Prior to using the chi square test, there are certain requirements that must be met.
  - The data must be in the form of frequencies counted in each of a set of categories. Percentages cannot be used.
  - The total number observed must be exceed 20.
- The expected frequency under the H0 hypothesis in any one fraction must not normally be less than 5.
- All the observations must be independent of each other. In other words, one observation must not have an influence upon another observation.(independent :like smoker, non smoker   male female)

Application of chi-square test

- Testing independence (or Association)
- Testing for homogeneity
- Testing of goodness-of-fit

Chi-square test

Objective : Smoking is a risk factor for MI
Null Hypothesis: Smoking does not cause MI

Smokers & non smokers & MI & not MI should be seperated in the table.
It's one of requirements, the last point in the previous slide.

	D (MI)	No D(No MI)	Total
Smokers	29	21	50
Non-smokers	16	34	50
Total	45	55	100

29 O / E	21 O / E
16 O / E	34 O / E

29 O / E 22.5	21 O / E 27.5
16 O / E 22.5	34 O / E 27.5

$$\text{MI\textbackslash Smoker} = E = \frac{45\times50}{100} = 22.5 \qquad \text{Non MI\textbackslash Smoker} = E = \frac{55\times50}{100} = 27.5$$

$$\text{MI\textbackslash Non Smoker} = E = \frac{45\times50}{100} = 22.5 \qquad \text{Non MI \textbackslash Non Smoker} = E = \frac{55\times50}{100} = 27.5$$

First, u have the Observed freq schedule,
Second, calculate Expected freq.
Third, Calculate the chi-square. $\chi^2 = \sum\frac{(O-E)^2}{E}$
Fourth, find the degree of freedom.
Last take a look at the table (critical value, just like z-table) so u can decide if null hypothesis is accepted or rejected

Chi-square

- Degrees of Freedom
  df = (r-1) (c-1)
  = (2-1) (2-1) =1

R= rows صفوف
C=columns اعمدة

في الاختبار اذا جاء سؤال كم عدد الصفوف والاعمدة في هذا الجدول table 2*2: ما نحسب التوتال

- Critical Value (Table A.6) = 3.8
- x^2= 6.84
- Calculated value(6.84) is greater than critical (table) value (3.84) at 0.05 level with 1 d.f.f
- Hence we reject our Ho and conclude that there is highly statistically significant association between smoking and MI.

Association between Diabetes and Heart Disease?

Background:
Contradictory opinions:
1. A diabetic's risk of dying after a first heart attack is the same as that of someone without diabetes. There is no association between diabetes and heart disease.

vs.

2. Diabetes takes a heavy toll on the body and diabetes patients often suffer heart attacks and strokes or die from cardiovascular complications at a much younger age.

- So we use hypothesis test based on the latest data to see what's the right conclusion.
- There are a total of 5167 patients, among which 1131 patients are non-diabetics and 4036 are diabetics. Among the non-diabetic patients, 42% of them had their blood pressure properly controlled (therefore it's 475 of 1131). While among the diabetic patients only 20% of them had the blood pressure controlled (therefore it's 807 of 4036).

- Data

	Controlled	Uncontrolled	Total
Non-diabetes	475	656	1131
Diabetes	807	3229	4036
Total	1282	3885	5167

CONT. Association between Diabetes and Heart Disease?

Data:
Diabetes: 1=Not have diabetes, 2=Have Diabetes
Control: 1=Controlled, 2=Uncontrolled

DIABETES * CONTROL Crosstabulation

Count

		CONTROL		Total
		1.00	2.00	
DIABETES	1.00	475	656	1131
	2.00	807	3229	4036
Total		1282	3885	5167

DIABETES * CONTROL Crosstabulation

			CONTROL		Total
			1.00	2.00	
DIABETES	1.00	Count	475	656	1131
		% within DIABETES	42.0%	58.0%	100.0%
		% within CONTROL	37.1%	16.9%	21.9%
		% of Total	9.2%	12.7%	21.9%
	2.00	Count	807	3229	4036
		% within DIABETES	20.0%	80.0%	100.0%
		% within CONTROL	62.9%	83.1%	78.1%
		% of Total	15.6%	62.5%	78.1%
Total		Count	1282	3885	5167
		% within DIABETES	24.8%	75.2%	100.0%
		% within CONTROL	100.0%	100.0%	100.0%
		% of Total	24.8%	75.2%	100.0%

Hypothesis test:
H_0: There is no association between diabetes and heart disease. (or) Diabetes and heart disease are independent.

H_A : There is an association between diabetes and heart disease. (or) Diabetes and heart disease are dependent.

- Assume a significance level of 0.05

SPSS Output

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	229.268[b]	1	.000		
Continuity Correction[a]	228.091	1	.000		
Likelihood Ratio	212.149	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	229.224	1	.000		
N of Valid Cases	5167				

a. Computed only for a 2x2 table

b. 0 cells (.0%) have expected count less than 5. The minimum expected count is 280.62.

- The computer gives us a Chi-Square Statistic of 229.268
- The computer gives us a p-value of .000 (<0.0001). p-value is smaller than alpha 0.05 which means there's significant association.
- Because our p-value is less than alpha, we would reject the null hypothesis.
- There is sufficient evidence to conclude that there is an association between diabetes and heart disease.

Chi- square test

Find out whether the gender is equally distributed among each age group

Age				
Total	45<	30-45	30>	Gender
120	(30) 40	(30) 20	(60) 60	Male
80	(20) 10	(20) 30	(40) 40	Female
200	50	50	100	total

Test for Homogeneity (Similarity)

To test similarity between frequency distribution or group. It is used in assessing the similarity between non-responders and responders in any survey

Age (yrs)	Responders	Non-responders	Total
<20	76 (82)	20 (14)	96
20 – 29	288 (289)	50 (49)	338
30-39	312 (310)	51 (53)	363
40-49	187 (185)	30 (32)	217
>50	77 (73)	9 (13)	86
Total	940	160	1100

❖ Fisher's exact test:

Study variable:	(Qualitative (Categorical
Outcome variable:	(Qualitative(Categorical
Comparison:	two proportions
Sample size:	X< 30

the difference here is the sample size is small

Example

The following data relate to suicidal feelings in samples of psychotic and neurotic patients:

	Psychotics	Neurotics	Total
Suicidal feelings	2	6	8
No suicidal feelings	18	14	32
Total	20	20	40

Example

The following data compare malocclusion of teeth with method of feeding infants.

	Normal teeth	Malocclusion
Breast fed	4	16
Bottle fed	1	21

Fisher's Exact Test:

The method of Yates's correction was useful when manual calculations were done. Now different types of statistical packages are available. Therefore, it is better to use Fisher's exact test rather than Yates's correction as it gives exact result.

$$Fisher's\ Exact\ Test = \frac{R_1!R_2!C_1!C_2!}{n!a!b!c!d!}$$

no need to know it, however interpretations is the same

What to do when we have a paired samples and both the exposure and outcome variables are qualitative variables (Binary).

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❖ Macnemar's test: (for paired samples)

paired: dependent variables.

Study variable:	(Qualitative (Categorical
Outcome variable:	(Qualitative(Categorical
Comparison:	two proportions
Sample size:	Any

Problem

- A researcher has done a matched case-control study of endometrial cancer (cases) and exposure to conjugated estrogens (exposed).
- In the study cases were individually matched 1:1 to a non-cancer hospital-based control, based on age, race, date of admission, and hospital.

Situation:

- Two paired binary variables that form a particular type of 2 x 2 table
- e.g. matched case-control study or cross-over trial

- **Data**

	controls	Cases	Total
Exposed	19	55	74
Not exposed	164	128	292
Total	183	183	366

- can't use a chi-squared test - observations are not independent - they're paired.
- we must present the 2 x 2 table differently
- each cell should contain a count of the number of pairs with certain criteria, with the columns and rows respectively referring to each of the subjects in the matched pair
- the information in the standard 2 x 2 table used for unmatched studies is insufficient because it doesn't say who is in which pair - ignoring the matching

	controls		
cases	Not exposed	Exposed	Total
Exposed	43	12	55
Not exposed	121	7	128
Total	164	19	183

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Cont.

We construct a matched 2 x 2 table:

	controls		
cases	Not exposed	Exposed	Total
Exposed	f	e	e+f
Not exposed	h	g	g+h
Total	f+h	e+g	n

Formula

The odds ratio is: f/g

The test is: $$X^2 = \frac{(|f - g| - 1)^2}{f + g}$$

Compare this to the χ^2 distribution on 1 df

$$X^2 = \frac{(|43 - 7| - 1)^2}{43 + 7} = \frac{1225}{50} = 24.5$$

P <0.001, Odds Ratio = 43/7 = 6.1

$p_1 - p_2$ = (55/183) – (19/183) = 0.197 (20%)

s.e.($p_1 - p_2$) = 0.036

95% CI: 0.12 to 0.27 (or 12% to 27%)

- Degrees of Freedom
  df = (r-1) (c-1)
  = (2-1) (2-1) =1
- Critical Value (Table A.6) = 3.84

- x^2= 25.92
- Calculated value(25.92) is greater than critical (table) value (3.84) at 0.05 level with 1 d.f.f
- Hence we reject our Ho and conclude that there is highly statistically significant association between Endometrial cancer and Estrogens.

Two-tailed critical ratios of χ^2

Degrees of freedom df	.10	.05	.02	.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.341
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.168	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141

Stata Output

```
                 | Controls                    |
Cases            |  Exposed    Unexposed     |       Total
-----------------+-------------------------+------------
         Exposed |       12           43     |          55
       Unexposed |        7          121     |         128
-----------------+-------------------------+------------
           Total |       19          164     |         183

McNemar's chi2(1) =      25.92    Prob > chi2 = 0.0000
Exact McNemar significance probability       = 0.0000

Proportion with factor
         Cases       .3005464
         Controls    .1038251     [95% Conf. Interval]
                     ---------    --------------------
         difference  .1967213     .1210924    .2723502
         ratio       2.894737     1.885462    4.444269
         rel. diff.  .2195122     .1448549    .2941695

         odds ratio  6.142857     2.739772    16.18458  (exact)
```

In Conclusion,

When both the study variables and outcome variables are categorical (Qualitative):
Apply
(i) Z-test (single & two proportions)
(i) Chi square test (two & more proportions)
(ii) Fisher's exact test (two proportions--Small samples)
(iii) Macnemar's test (two proportions of paired samples)