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## LECTURE OBJECTIVES



By the end of this lecture, I am able to:Able to understand how to summarize the data
Able to understand how to measure the variability of the data.
Able to use and interpret appropriately the different summary and variability measures

## Investigation

## Data

 PresentationTabulation
Diagrams
Graphs

## Descriptive Statistics

Measures of Location Measures of Dispersion Measures of Skewness \& Kurtosis

Inferential Statistics

Estimation Testing
Point estimate
Interval estimate Hypothesis

## Inferential statistics

Univariate analysis
Multivariate analysis

## Summary \& Variability Measures



## Quartiles

## Measures of Central Tendency

A statistical measure that identifies a single score as representative for an entire distribution. The goal of central tendency is to find the single score that is most typical or most representative of the entire group.

## Three common measures of central tendency:

The Mean
sum/total (average)

The Median
middle number of all data

## The Mean

Calculate the mean of the following data:
15432
Sum the scores $(\Sigma X)$ :
$1+5+4+3+2=15$
Divide the sum $(\Sigma X=15)$ by the number of
scores $(N=5): 15 / 5=3$
Mean affected by extreme value $=\mathrm{X}=3$

## Mean (Arithmetic Mean)

The most common measure of central tendency
Affected by extreme values (outliers)*
extreme value mean for example is the student mark is between 10-12
but there is 4 student get full mark or the opposite if there is 3 student get
zero in the exam


## The Median

The median is simply another name for the 50th percentile
It is the score in the middle; half of the scores are larger than the median and half of the scores are smaller than the median

## Calculate the Median

Conceptually, it is easy to calculate the median
Sort the data from highest to lowest
Find the score in the middle
middle $=(N+1) / 2$
If $N$, the number of scores is even, the median is the average of the middle two scores

## Median Examples

## Example 1:

- What is the median of the following scores:

241819421612
Sort the scores:
422419181612
Determine the middle score:
middle $=(N+1) / 2=(6+1) / 2=3.5$
Median = average of 3rd and 4th scores:
$(19+18) / 2=18.5$
-first we arrange the number from high to low
-N= sample size
-here the median because the sample size is
double (even) we take the average of the
middle two numbers
'the median not affected by extreme values'

## Example 2:

- What is the median of the following scores:

$$
1081415733812109
$$

Sort the scores:
1514121010988733
Determine the middle score:
middle $=(N+1) / 2=(11+1) / 2=6$
Middle score $=$ median $=9$
the only difference here is that the sample size is
single (odd) so after we rearrange them we take the
middle one

## Median

In an ordered array, the median is the "middle" number
If n or N is odd, the median is the middle number
If n or N is even, the median is the average of the two middle numbers (example if $\mathrm{n}=42$ then the
median is the average of the 21st and 22nd values)


Not affected by extreme values

## Measures of Central Tendency



- 12345678920

| 6.5 | 5 |
| :--- | :--- |

- 123456789
5.5
5.5

Mean is the most frequently used but is sensitive to extreme scores

## Mode

mainly applied outside
educational purpose

1. A Value that occurs most often
2. Not affected by extreme values
3. Used for either numerical or categorical(nominal)data
4. There may be no mode (sample size isis low for eg when you take the mark for 3 student only there will be no repetid)
5. There may be several modes



No Mode

## The Shape of Distributions

## Symmetrical

## Skewed

shift to one side

Depending on whether there are more frequencies at one end of the distribution than the other.

## Symmetrical Distributions

A distribution is symmetrical if the frequencies at the right and left tails of the distribution are identical, so that if it is divided into two halves, each will be the mirror image of the other.

In a symmetrical distribution the mean, median, and mode are identical.

Bell-Shaped (also known as symmetric" or "normal")


## Skewed:

Few extreme values on one side of the distribution or on the other.

1. Positively skewed distributions: distributions which have few extremely high values (Mean>Median)
2. Negatively skewed distributions: distributions which have few extremely low values(Mean<Median)

## Skewed

"as you go to the right the number increase

- positively (skewed to the right) it tails off toward larger values
pic1:for example the mark of medicine only
$5 \%$ of the student will get A+ but the majority will be less than that ,the PIC reflet the majority of student in the left(less number)and the student who get high mark on the right
- negatively (skewed to the left) it tails off toward smaller values pic2 :for example the student mark in research course the majority of the student get A A+ so they will be on the right of the chart



## Choosing a Measure of Central tendency

IF variable is Nominal
like gender

IF variable is Ordinal

IF variable is Interval-Ratio and distribution is Symmetrical

Mode or Median (or both)

Example

# (1) $7,8,9,10,11 \quad n=5, \sum \dot{x}=45, x=45 / 5=9$ <br> (2) $3,4,9,12,15 \quad \mathrm{n}=5, \Sigma \mathrm{x}=45, \bar{x}=45 / 5=9$ <br> (3) $1,5,9,13,17 \quad \mathrm{n}=5 \geqslant \mathrm{x}=45, \quad \bar{x}=45 / 5=9$ <br> S.D. : (1) 1.58 (2) 4.74 (3) 6.32 

## Measures of Dispersion

Measures of dispersion summarize differences in the data, how the numbers differ from one another.

Series I: 70707070707070707070 no variability
Series II: 66676869707071727374 Small variability
Series III: 1195060708090100110120 High variability

## Measures of Variability

A single summary figure that describes the spread of observations within a distribution.


Standard Deviation

Rough measure of the average amount by which observations deviate from the mean. The square root of the variance.

## Variability Example: Range

Marks of students:
$52,76,100,36,86,96,20,15,57,64,64,80,82,83,30$, $31,31,31,32,37,38,38,40,40,41,42,47,48,63,63$, 72, 79, 70, 71, 89

Range:
100-15 = 85

## Quartiles

> Q1, Q2, Q3

Divides ranked scores into four equal parts


## Quartiles: $Q_{1}=\frac{n+1}{4}$,



Inter quartile :

$$
\mathrm{IQR}=\mathrm{Q}_{3-} \mathrm{Q}_{1}
$$

## Interquartile Range

The interquartile range is Q3-Q1
$50 \%$ of the observations in the distribution are in the inter quartile range.
The following figure shows the interaction between the quartiles, the median and the interquartile range.

Figure 3.6
The midefie haif of the observertions in a irequency cistribution li申 withoin the interquartile range


## Percentiles and Quartiles

Maximum is 100th percentile: $100 \%$ of values lie at or below the maximum
Median is 50th percentile: $50 \%$ of values lie at or below the median
Any percentile can be calculated. But the most common are 25th (1st Quartile) and 75th (3rd Quartile)

## Locating Percentiles in a Frequency Distribution

A percentile is a score below which a specific percentage of the distribution falls(the median is the 50th percentile.

The 75th percentile is a score below which $75 \%$ of the cases fall.
The median is the 50th percentile: $50 \%$ of the cases fall below it
Another type of percentile :The quartile lower quartile is 25 th percentile and the upper quartile is the 75th percentile

| 25th percentile | NUMBER OF CHILDREN |  |  |  |  |  | $\text { / } \begin{aligned} & 25 \% \\ & \text { included } \\ & \text { here } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Frequency | Percent | Valid Percent | Cumulative Percent |  |
|  | Valid | 0 | 260 | 26.6 | 26.6 | 26.6 |  |
| 50th percentile |  | 1 | 161 | 16.4 | 16.5 | 43.1 | $\text { 50\% } \begin{aligned} & 50 \\ & \text { included } \\ & \text { here } \end{aligned}$ |
|  |  | 2 | 260 | 26.6 | 26.6 | 69.7 |  |
|  |  | 3 | 155 | 15.8 | 15.9 | 85.6 |  |
| 80th percentile |  | 4 | 70 | 7.2 | 7.2 | 92.7 |  |
|  |  | 5 | 31 | 3.2 | 3.2 | 95.9 | 80\% <br> included |
|  |  | 6 | 21 | 2.1 | 2.1 | 98.1 |  |
|  |  | 7 | 11 | 1.1 | 1.1 | 99.2 |  |
|  |  | EIGHT OR MORE | 8 | . 8 | . 8 | 100.0 | here |
|  |  | Total | 977 | 99.8 | 100.0 |  |  |
|  | Missing | NA | 2 | . 2 |  |  |  |
|  | Total |  | 979 | 100.0 |  |  |  |

## Variance

Deviations of each observation from the mean, then averaging the sum of squares of these deviations.

## Standard Deviation

## " ROOT- MEANS-SQUARE-DEVIATIONS"

To "undo" the squaring of difference scores, take the square root of the variance.
Return to original units rather than squared units.

## Quantifying Uncertainty

Standard deviation: measures the variation of a variable in the sample.
-Technically,

$$
s=\sqrt{\frac{1}{N-1} \sum_{i=1}^{N}\left(x_{i}-\bar{x}\right)^{2}}
$$

## Example

$$
\text { Data: } X=\{6,10,5,4,9,8\} ; \quad N=6
$$

| $X$ | $X-\bar{X}$ | $(X-\bar{X})^{2}$ |
| :--- | :---: | :---: |
| 6 | -1 | 1 |
| 10 | 3 | 9 |
| 5 | -2 | 4 |
| 4 | -3 | 9 |
| 9 | 2 | 4 |
| 8 | 1 | 1 |

Mean:

$$
\bar{X}=\frac{\sum_{N} X}{N}=\frac{42}{6}=7
$$

Variance:

$$
s^{2}=\frac{\sum(\bar{X}-X)^{2}}{N}=\frac{28}{6}=4.67
$$

Standard Deviation:
$s=\sqrt{s^{2}}=\sqrt{4.67}=2.16$

Interpretation: All 6 values on average are deviating by
2.16. On average each student is different from other by
2.16 .

## Calculation of Variance \& Standard Deviation

Using the deviation \& computational method to calculate the variance and standard deviation

## Calculation of Variance \& Standard Deviation

## Example

$3,4,4,4,6,7,7,8,8,9$
Given $n=10 ;$ Sum $=60 ;$ Mean $=6$
$S=\sqrt{\frac{\sum(X-\bar{X})^{2}}{n}}$
$S=\sqrt{\frac{(3-6)^{2}+(4-6)^{2}+(4-6)^{2}+(4-6)^{2}+(6-6)^{2}+(7-6)^{2}+(7-6)^{2}+(8-6)^{2}+(8-6)^{2}+(9-6)^{2}}{10}}$
$S=\sqrt{\frac{40}{10}}=2.0 ;$ var iance $=4$


## Descriptive Statistics

Variable: Age

Anderson-Darling Normality Test

| A-Squared: | 0.962 |
| :--- | ---: |
| P-Value: | 0.014 |
|  |  |
| Mean | 36.4500 |
| StDev | 15.7356 |
| Variance | 247.608 |
| Skewness | 0.679626 |
| Kurtosis | $8.51 \mathrm{E}-02$ |
| N | 60 |
|  |  |
| Minimum | 11.0000 |
| 1st Quartile | 25.0000 |
| Median | 31.5000 |
| 3rd Quartile | 46.7500 |
| Maximum | 79.0000 |

## 95\% Confidence Interval for Mu <br> 32.3851 <br> 40.5149

95\% Confidence Interval for Sigma 13.3380
19.1921

95\% Confidence Interval for Median 28.0000
42.0000

## WHICH MEASURE TO USE?

DISTRIBUTION OF DATA IS SYMMETRICAL ----> USE MEAN \& S.D.,

DISTRIBUTION OF DATA IS SKEWED ----> USE MEDIAN \& QUARTILES

## Flow chart of commonly used descriptive statistics and graphical illustrations <br> - Descriptive statistics

## Exploring data

- Categorical data
> Frequency
> Percentage (Row, Column or Total)
- Continuous data: Measure of location
> Mean
> Median
- Continuous data: Measure of variation
- Standard deviation
> $\quad$ Range (Min, Max)
> Inter-auartile range (LQ, UQ)
- Categorical data
> Bar chart
> Clustered bar charts (two categorical variables)
Pie charts
- Continuous data
- Graphical illustrations
> Histogram (can be plotted against a categorical variable)
> Box \& Whisker plot (can be plotted against a categorical variable)
> Stem and Leaf plot
$>\quad$ Scatter plot (two continuous variables)

