Statistical tests to Observe the statistical significance of Quantitative variables

BY

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Learning Objectives:

- (1) Able to understand the factors to apply for the choice of statistical tests in analyzing the data .
- (2) Able to apply appropriately Z-test, student's t-test and Karl Pearson's Correlation Coefficient.
- (3) Able to interpret the findings of the analysis using these tests.

Choosing the appropriate Statistical test

- Based on the three aspects of the data
  - Types of variables
  - Number of groups being compared &
  - Sample size

Statistical Tests

Z-test:

Study variable: Qualitative

Outcome variable: Quantitative

Comparison: Sample mean with population mean & two sample means

Sample size: larger in each group(>30) & standard deviation is known

Student's t-test:

Study variable: Qualitative

Outcome variable: Quantitative

Comparison: sample mean with population mean; two means (independent samples); paired samples.

Sample size: each group <30 (can be used even for large sample size)

Example(Comparing Sample mean with Population mean):

- The education department at a university has been accused of “grade inflation” in medical students with higher GPAs than students in general.

- GPAs of all medical students should be compared with the GPAs of all other (non-medical) students.
  - There are 1000s of medical students, far too many to interview.
  - How can this be investigated without interviewing all medical students ?

What we know:

- The average GPA for ***all*** students is 2.70. This value is a **parameter.**

$\mu = 2.70$

- To the right is the statistical information for a random sample of medical students:

$\bar{X}$ =	3.00
s =	0.70
n =	117

Questions to ask:

- Is there a difference between the parameter (2.70) and the statistic (3.00)?
- Could the observed difference have been caused by random chance?
- Is the difference real (significant)?

1. The sample mean (3.00) is the same as the pop. mean (2.70).
   - The difference is trivial and caused by random chance.

2. The difference is real (significant).
   - Medical students are different from all other students.

Step 1: Make Assumptions and Meet Test Requirements

- Random sampling
  - Hypothesis testing assumes samples were selected using random sampling.
  - In this case, the sample of 117 cases was randomly selected from all medical students.
- Level of Measurement of GPA is a Ratio scale
  - so the mean is an appropriate statistic.
- Sampling Distribution is normal in shape
  - This is a “large” sample ($n \geq 100$).

Step 2 State the Null Hypothesis

- H_0: μ = 2.7 (in other words, H_0: $\bar{X}$ = μ)
  - You can also state H_o: No difference between the sample mean and the population parameter
  - (In other words, the sample mean of 3.0 really the same as the population mean of 2.7 – the difference is not real but is due to chance.)
  - The sample of 117 comes from a population that has a GPA of 2.7.
  - The difference between 2.7 and 3.0 is trivial and caused by random chance.

Step 2 (cont.) State the Alternat Hypothesis

- H_1: μ≠2.7 (or, H_0: $\bar{X} \neq \mu$)
  - Or H_1: There is a difference between the sample mean and the population parameter
  - The sample of 117 comes a population that *does not* have a GPA of 2.7. In reality, it comes from a different population.
  - The difference between 2.7 and 3.0 reflects an actual difference between medical students and other students.
  - Note that we are testing whether the population the sample comes from is from a different population or is the same as the general student population.

Step 3 Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution= Z
  - Alpha (α) = .05
  - α is the indicator of “rare” events.
  - Any difference with a probability less than α is rare and will cause us to reject the H_0.

Step 3 (cont.) Select Sampling Distribution and Establish the Critical Region

- Critical Region begins at Z= $\pm$ 1.96
  - This is the critical Z score associated with $\alpha = .05$, two-tailed test.
  - If the obtained Z score falls in the Critical Region, or "the region of rejection," then we would reject the H_0.

When the Population σ is not known, use the following formula:

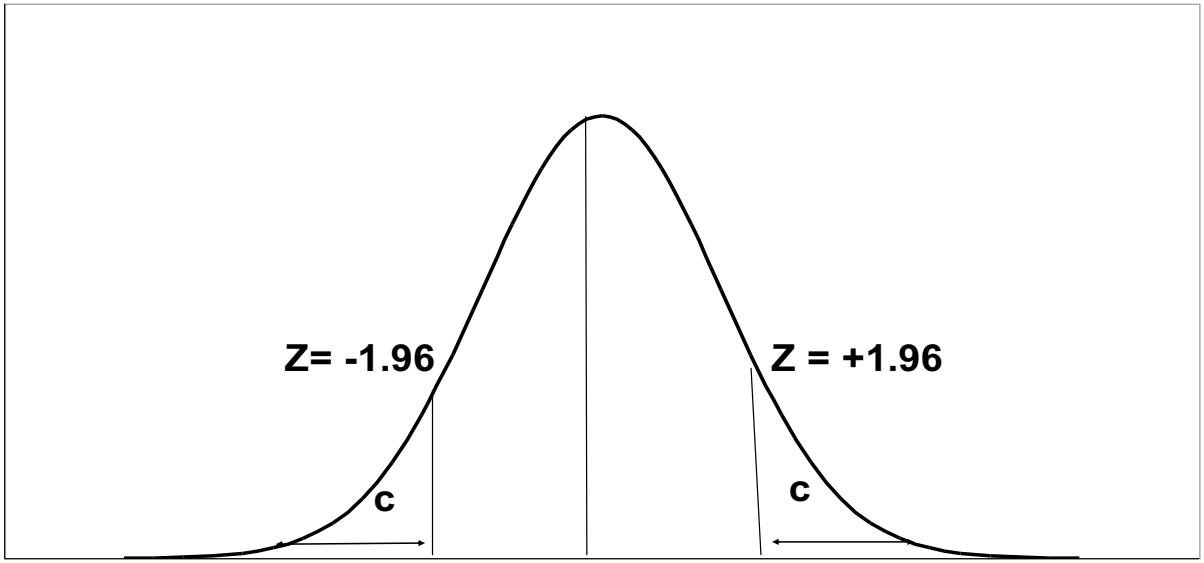
$$Z = \frac{\overline{X} - \mu}{s/\sqrt{n-1}}$$

Test the Hypotheses

$$Z = \frac{3.0 - 2.7}{.7 / \sqrt{117 - 1}} = 4.62$$

- Substituting the values into the formula, we calculate a Z score of 4.62.

Two-tailed Hypothesis Test



When α = .05, then .025 of the area is distributed on either side of the curve in area **(C)**

The .95 in the **middle section** represents **no significant difference** between the population and the sample mean.

The cut-off between the middle section and +/- .025 is represented by a **Z-value of +/- 1.96.**

Step 5 Make a Decision and Interpret Results

- The obtained Z score fell in the Critical Region, so we *reject* the H_0.
  - If the H_0 were true, a sample outcome of 3.00 would be unlikely.
  - Therefore, the H_0 is false and must be rejected.

- Medical students have a GPA that is significantly different from the non-medical students ($Z = 4.62$, $p < 0.05$).

Summary:

- The GPA of medical students is *significantly* different from the GPA of non-medical students.
- In hypothesis testing, we try to identify statistically significant differences that did not occur by random chance.
- In this example, the difference between the parameter 2.70 and the statistic 3.00 was large and unlikely ($p < .05$) to have occurred by random chance.

Example : Weight Loss for Diet vs Exercise

Did dieters lose more fat than the exercisers?

Diet Only:

sample mean = 5.9 kg

sample standard deviation = 4.1 kg

sample size = n = 42

standard error = $SEM_1 = 4.1/\sqrt{42} = 0.633$

Exercise Only:

sample mean = 4.1 kg

sample standard deviation = 3.7 kg

sample size = n = 47

standard error = $SEM_2 = 3.7/\sqrt{47} = 0.540$

measure of variability $= \sqrt{[(0.633)^2 + (0.540)^2]} = 0.83$

Example : Weight Loss for Diet vs Exercise

Step 1. Determine the null and alternative hypotheses.

Null hypothesis: No difference in average fat lost in population for two methods. Population mean difference is ***zero***.

Alternative hypothesis: There is a difference in average fat lost in population for two methods. Population mean difference is not ***zero***.

Step 2. Sampling distribution: Normal distribution (z-test)

Step 3. Assumptions of test statistic (sample size > 30 in each group)

Step 4. Collect and summarize data into a test statistic.

The sample mean difference = 5.9 – 4.1 = 1.8 kg
and the standard error of the difference is 0.83.

So the *test statistic:* $z = \dfrac{1.8 - 0}{0.83} = 2.17$

Example : Weight Loss for Diet vs Exercise

Step 5. Determine the *p*-value.

Recall the alternative hypothesis was two-sided.

p-value = 2 × [proportion of bell-shaped curve above 2.17]

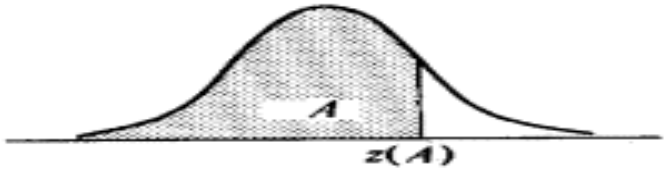
Z-test table => proportion is about 2 × 0.015 = 0.03.

Step 6. Make a decision.

The ***p*-value of 0.03 is less than or equal to 0.05**, so …

- If really no difference between dieting and exercise as fat loss methods, would see such an extreme result only 3% of the time, or 3 times out of 100.
- Prefer to believe truth does not lie with null hypothesis. We conclude that there is a ***statistically significant difference between average fat loss for the two methods***.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Student's t-test

1. **Test for single mean**
   Whether the sample mean is equal to the predefined population mean ?

2. **Test for difference in means**
   Whether the CD4 level of patients taking treatment A is equal to CD4 level of patients taking treatment B ?

3. **Test for paired observation**
   Whether the treatment conferred any significant benefit ?

Steps for test for single mean

1. **Questioned to be answered**

   Is the Mean SBP of the sample of 20 patients is 120?

   N=20, $\bar{x}$=135 , sd=5 , μ= 120

2. **Null Hypothesis**

   The mean SBP of 20 patients is 120. That is, The sample mean is equal to normal value (population mean).

3. **Test statistics** $t = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$ **--- t $_{(n-1)}$ df**

4. **Comparison with theoretical value**

   if tab t $_{(n-1)}$ < cal t $_{(n-1)}$ reject Ho,

   if tab t $_{(n-1)}$ > cal t $_{(n-1)}$ accept Ho,

5. **Inference**

t –test for single mean

- **Test statistics**

n=20, $\bar{x}$ =135, sd=5 , μ =120

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$ **= (135-120)/1.12 =13.39**

$t_\alpha = t_{.05,\ 19} = 2.093$

Accept H_0 if t < 2.093

Reject H_0 if t >= 2.093

Inference : We reject Ho, and conclude that the data is providing enough evidence, that the sample mean BP is significantly higher than the normal value.

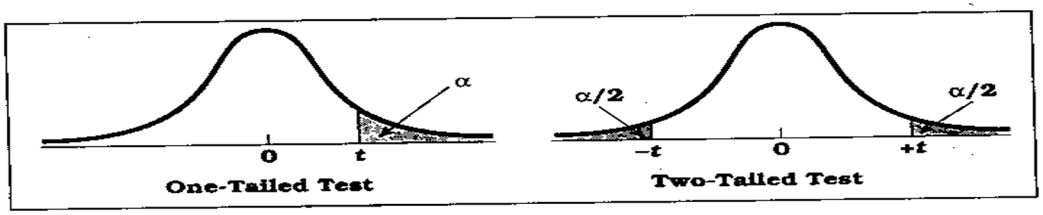


Table D.6 Percentage Points of the *t* Distribution (Source: The entries in this table were computed by the author.)

	Level of Significance for One-Tailed Test								
	.25	.20	.15	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Example

Sample of size 25 was selected from healthy population, their mean SBP =125 mm Hg with SD of 10 mm Hg . Another sample of size 17 was selected from the population of diabetics, their mean SBP was 132 mmHg, with SD of 12 mm Hg .

Test whether there is a significant difference in mean SBP of diabetics and healthy individual at 1% level of significance

t-Test (two independent means)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S^2p}{n_1} + \frac{S^2p}{n_2}}}$$

$\bar{X}_1$ = mean of the first group

$\bar{X}_2$ = mean of the second group

S^2p = pooled variance

$$S^2P = \frac{(n_1 - 1)S^2{}_1 + (n_2 - 1)S^2{}_2}{n_1 + n_2 - 2}$$

Critical t from table is detected

- at degree of freedom = $n_1 + n_2 - 2$
- level of significance 1% or 5%

Answer

$n_1 = 25$ $\quad \overline{X}_1 = 125$ $\quad S_1 = 12$

$n_2 = 17$ $\quad \overline{X}_2 = 132$ $\quad S_2 = 11$

State H0 $\quad H_0 : \mu_1 = \mu_2$

State H1 $\quad H_1 : \mu_1 \neq \mu_2$

Choose α $\quad \alpha = 0.01$

$$S^2P = \frac{(25-1)10^2 + (17-1)12^2}{25+17-2} = 117.6$$

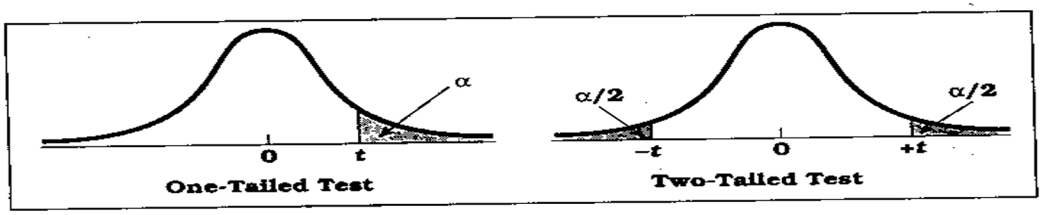
Answer

$$t = \frac{125 - 132}{\sqrt{\frac{117.6}{25} + \frac{117.6}{17}}} = -2.503$$

Critical t at df = 40 & 1% level of significance = 2.704

Decision:

Since the computed t is smaller than critical t so there is no significant difference between mean SBP of healthy and diabetic samples at 1 %.



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27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Paired t- test

Uses:

To compare the means of two paired samples.

Example, mean SBP before and after intake of drug.

Example

The following data represents the reading of SBP before and after administration of certain drug. Test whether the drug has an effect on SBP at 1% level of significance.

$$t = \frac{\bar{d}}{\frac{Sd}{\sqrt{n}}}$$

$$\bar{d} = \frac{\sum di}{n} = \text{mean difference}$$

$$Sd = \sqrt{\frac{\sum di^2 - \frac{(\sum di)^2}{n}}{n-1}}$$

di = difference (after-before)

Sd = standard deviation of difference

n = sample size

Critical t from table at df = n-1

Serial No.	SBP (Before)	SBP (After)
1	200	180
2	160	165
3	190	175
4	185	185
5	210	170
6	175	160

Answer

Serial No.	BP Before	BP After	di After-Before	di^2
1	200	180	-20	400
2	160	165	5	25
3	190	175	-15	225
4	185	185	0	0
5	210	170	-40	1600
6	175	160	-15	225
Total			-85	2475
			∑di	∑ di2

Answer

$$\bar{d} = \frac{\sum di}{n} = \frac{-85}{6} = -14.17$$

$$Sd = \sqrt{\frac{2475 - \frac{(-85)^2}{6}}{5}} = 15.942$$

Answer

$$\text{Computed t} = \frac{-14.17}{\frac{15.942}{\sqrt{6}}} = -2.17$$

Critical t at df = 6-1 = 5 and 1% level of significance

= 4.032

Decision:

Since t is < critical t so there is no significant difference between mean SBP before and after administration of drug at 1% Level.

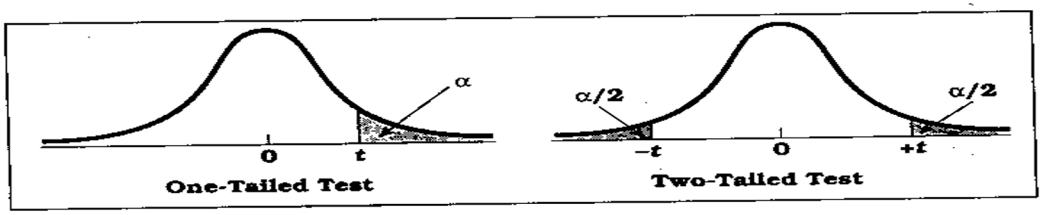


Table D.6 Percentage Points of the *t* Distribution (Source: The entries in this table were computed by the author.)

	Level of Significance for One-Tailed Test								
	.25	.20	.15	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test								
df	.50	.40	.30	.20	.10	.05	.02	.01	.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	63.662
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.496
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.390
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Z- value & t-Value

"Z and t" are the measures of:
How difficult is it to believe the null hypothesis?

High z & t values

Difficult to believe the null hypothesis - accept that there is a real difference.

Low z & t values

Easy to believe the null hypothesis - have not proved any difference.

Karl Pearson Correlation Coefficient

Working with two variables (parameter)

As Age ↑ BP ↑

As Height ↑ Weight ↑

As Age ↑ Cholesterol ↑

As duration ↑
of HIV CD4 CD8 ↓

A number called the **correlation** measures both the direction and strength of the linear relationship between two related sets of quantitative variables.

Correlation Contd….

- Types of correlation –
- Positive – Variables move in the same direction

- Examples:
- Height and Weight
- Age and BP

Correlation contd…

- Negative Correlation
- Variables move in opposite direction

- Examples:
- Duration of HIV/AIDS and CD4 CD8
- Price and Demand
- Sales and advertisement expenditure

Correlation contd…..

- Measurement of correlation

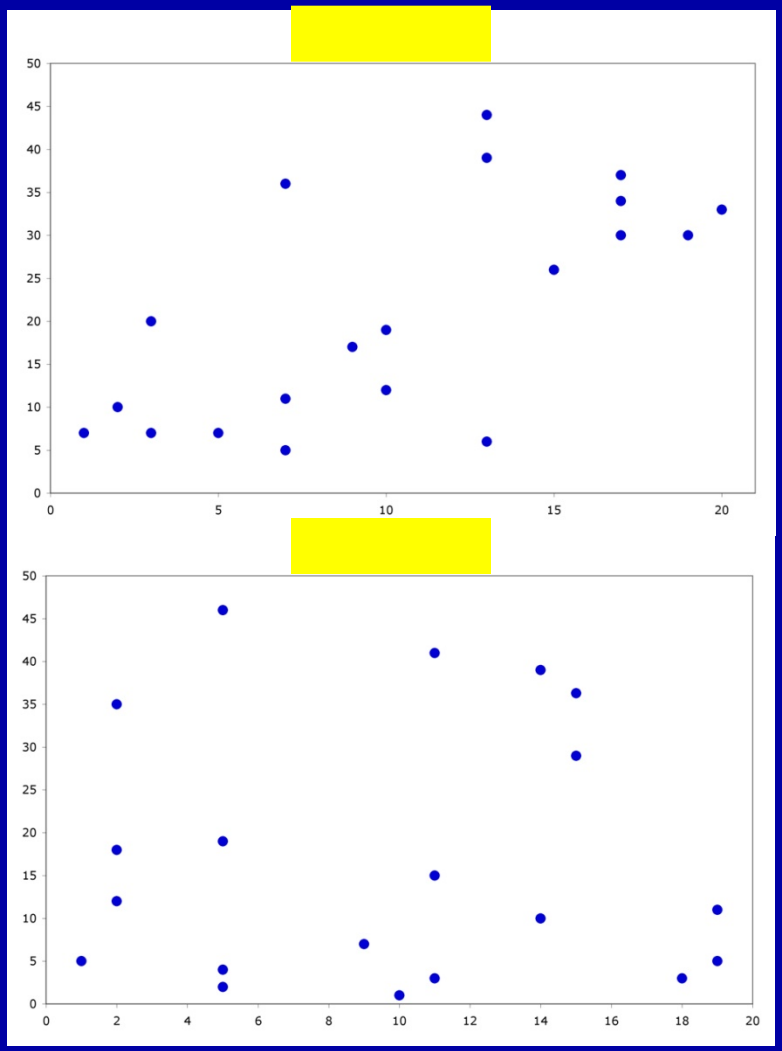
1. **Scatter Diagram**
2. **Karl Pearson's coefficient of Correlation**

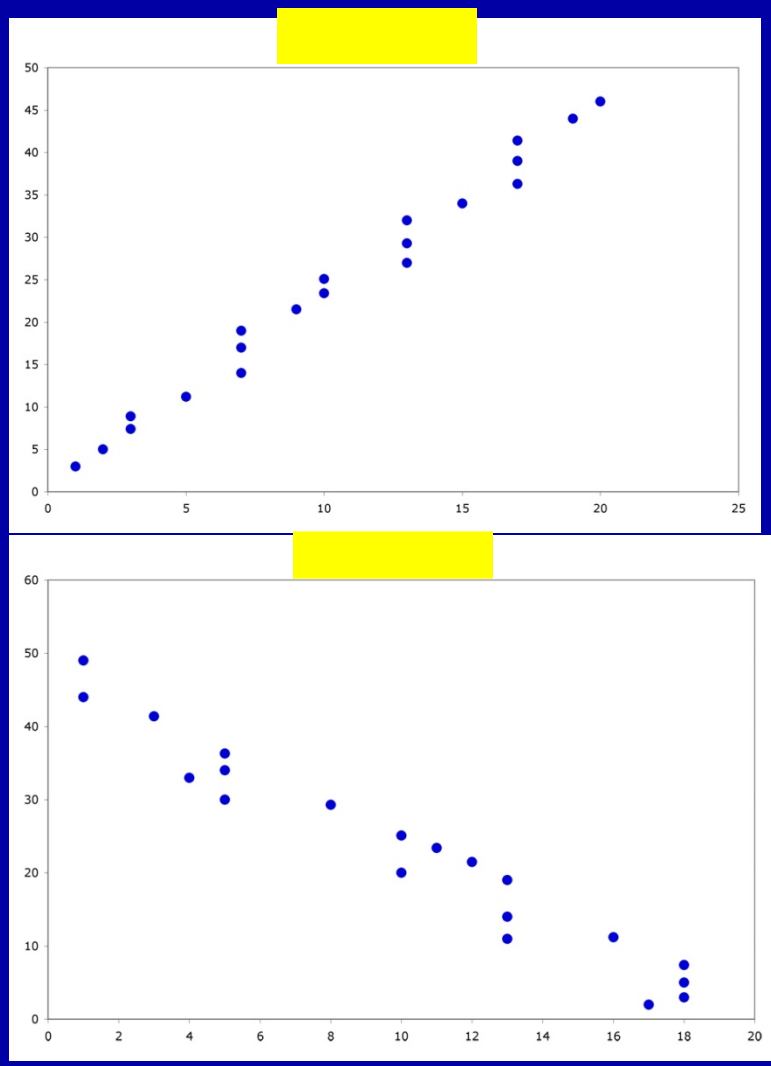
Graphical Display of Relationship

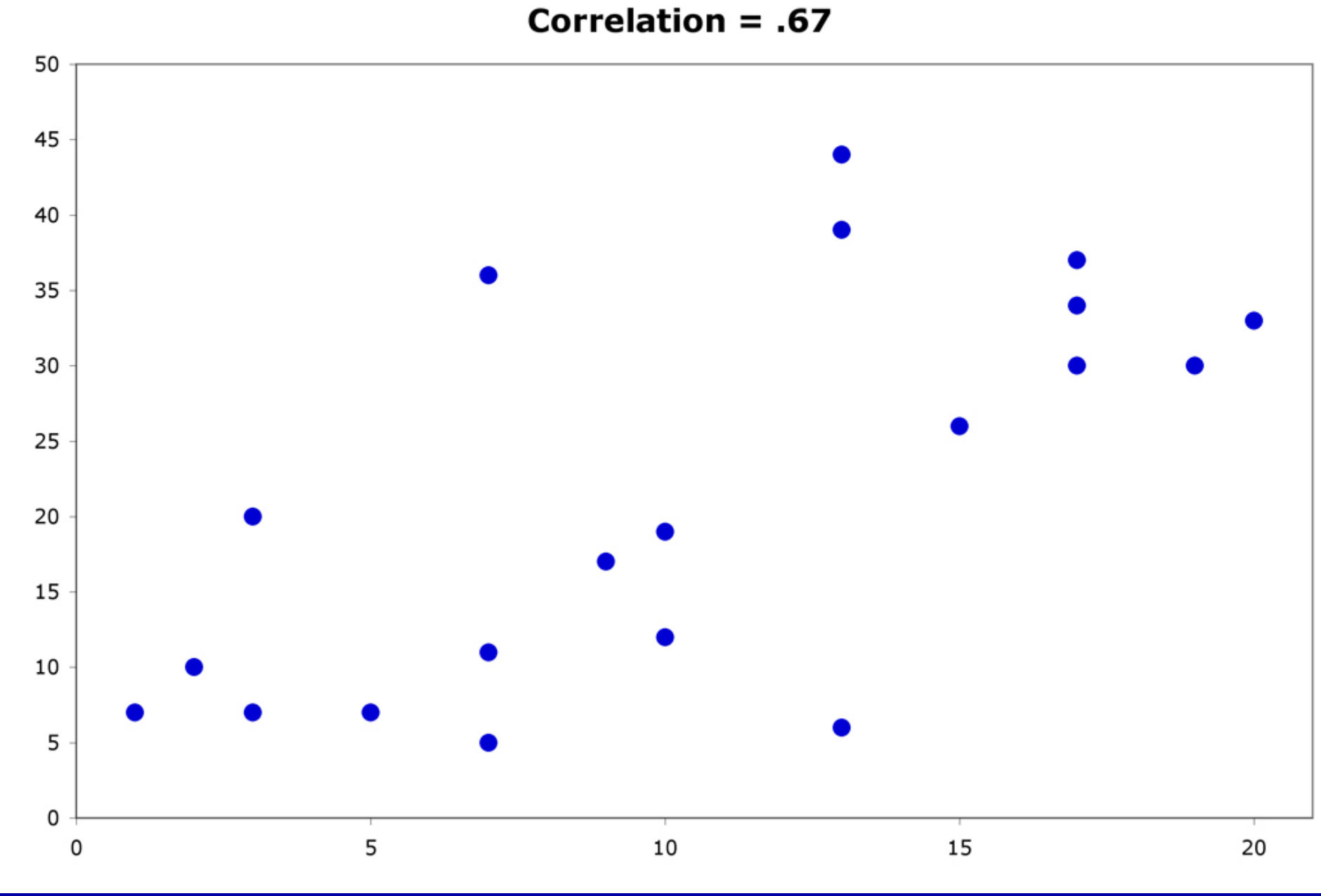
- Scatter diagram
- Using the axes
  - X-axis horizontally
  - Y-axis vertically
  - Both axes meet: origin of graph: 0/0
  - Both axes can have different units of measurement
  - Numbers on graph are (x,y)

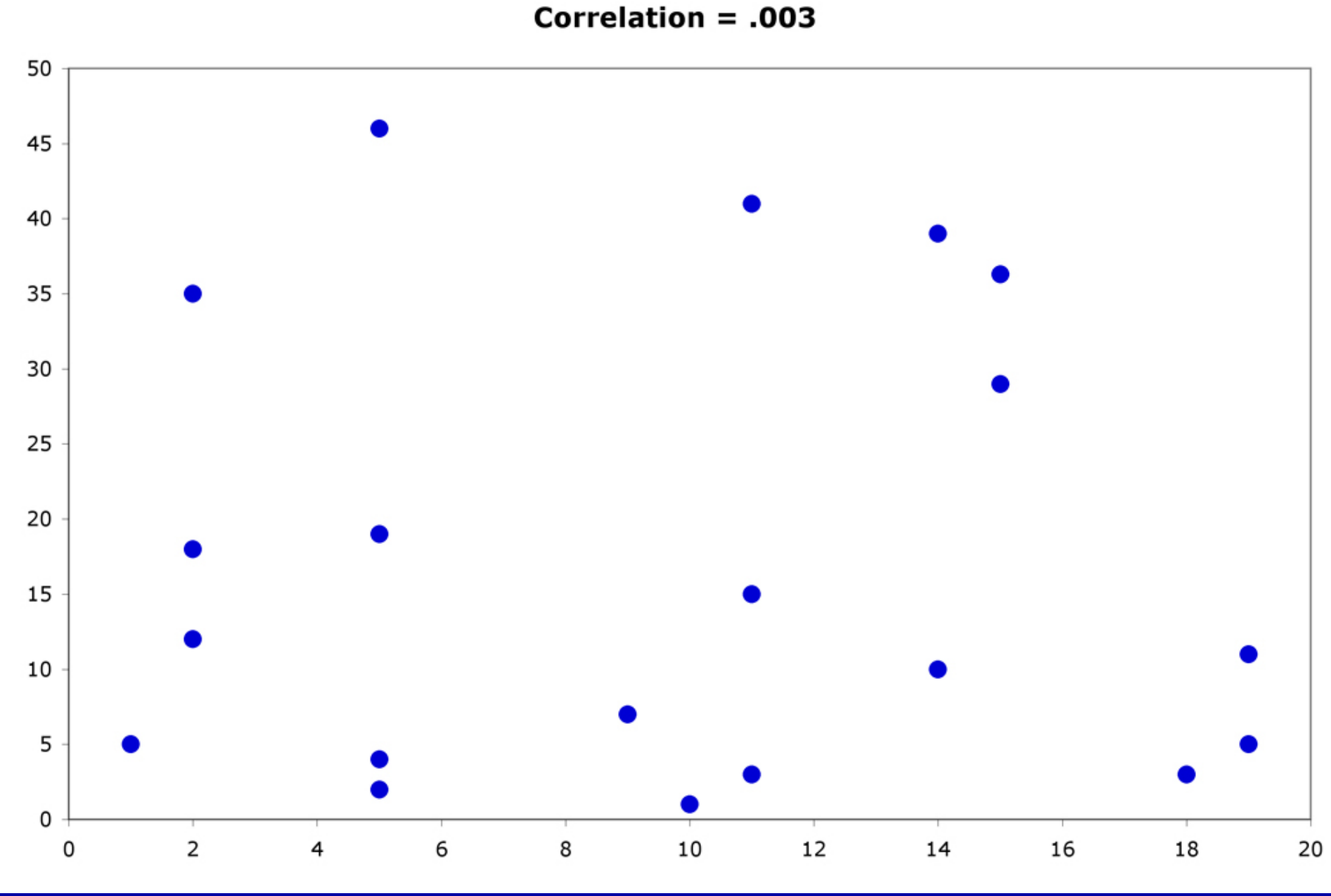
Guess the Correlations:

.67 .993 .003 -.975






Correlation = .67
50
45
40
35
30
25
20
15
10
5
0
0
5
10
15
20


Correlation = .003
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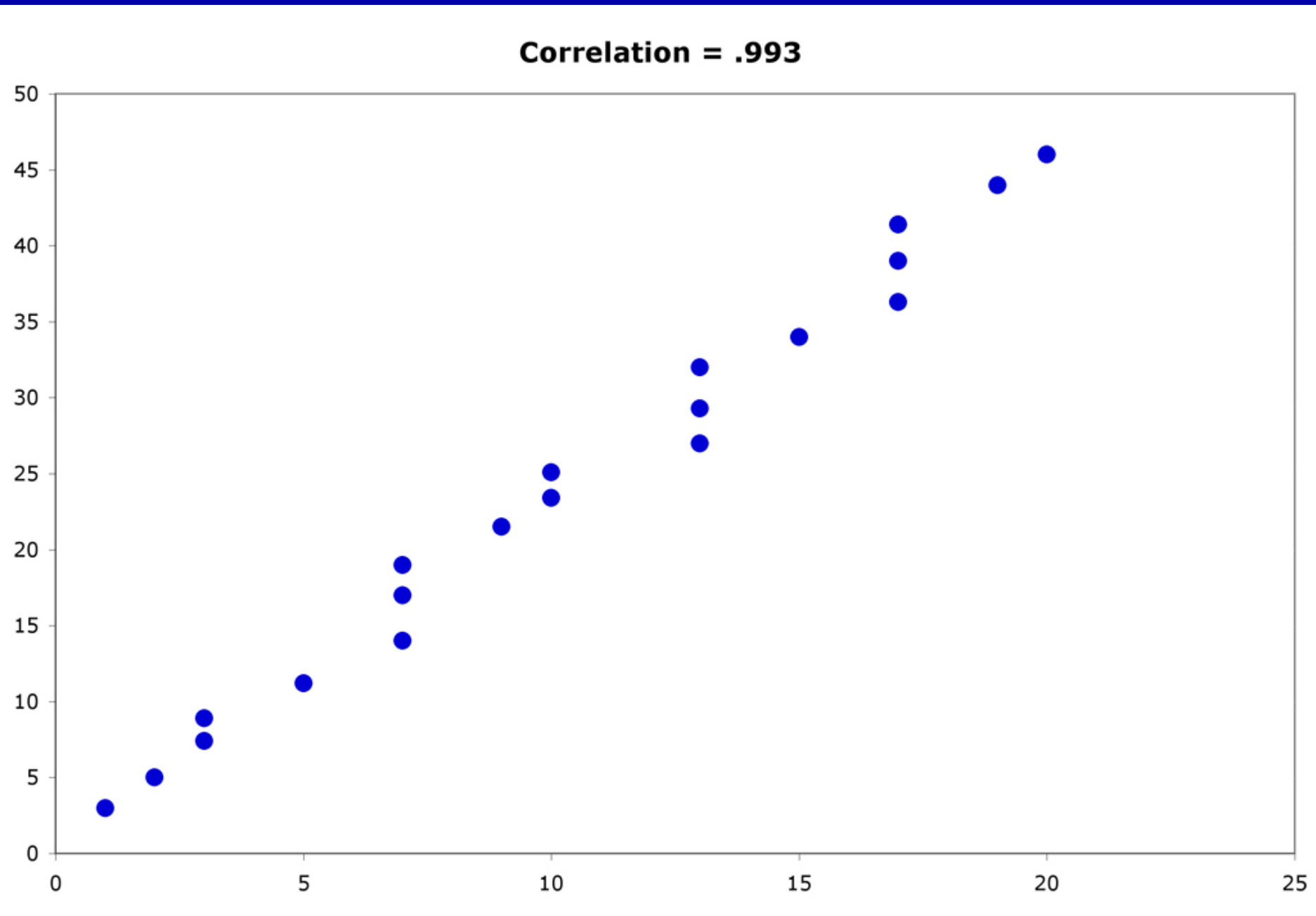
45
40
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
18
20


Correlation = .993
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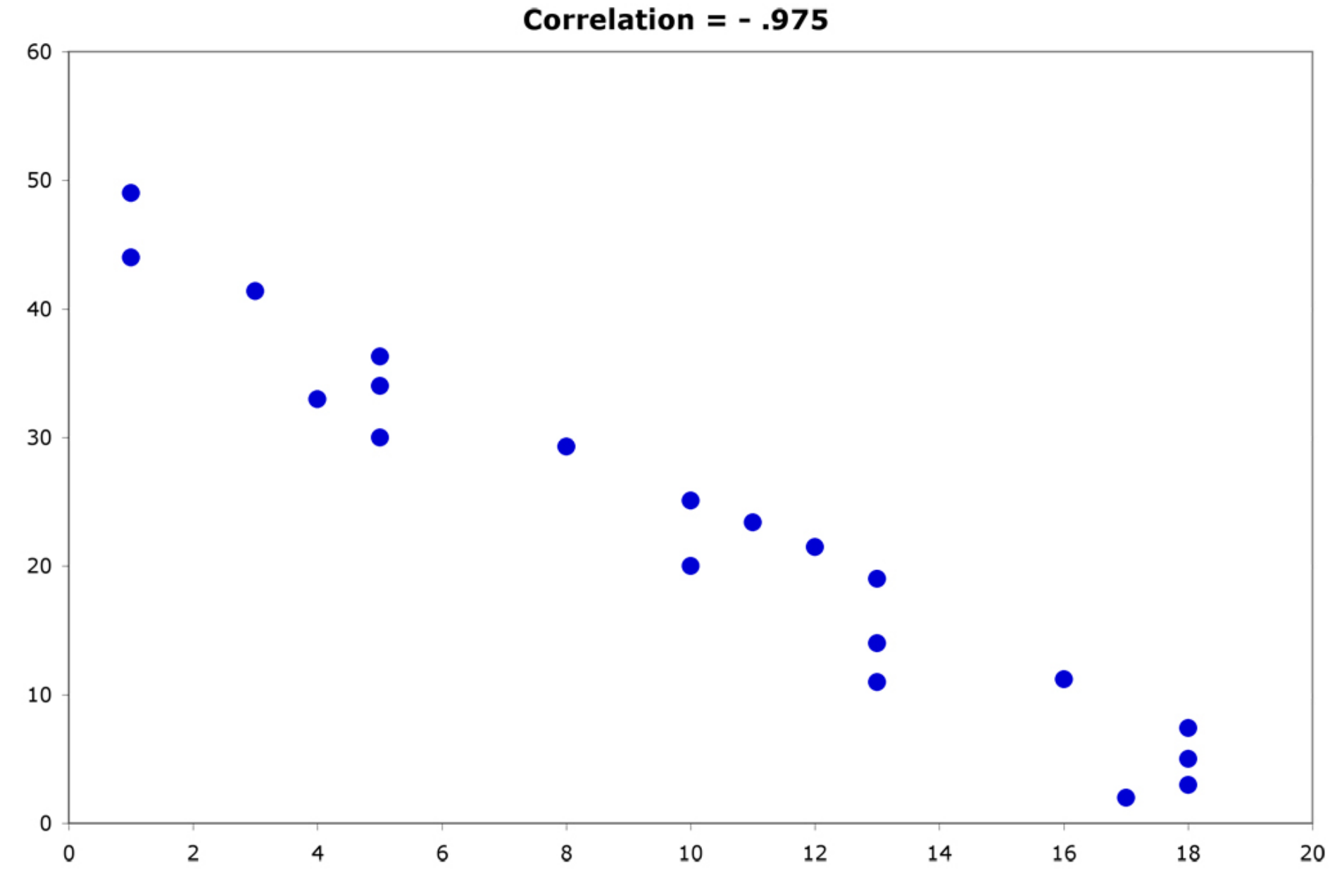
45
40
35
30
25
20
15
10
5
0
0
5
10
15
20
25


Correlation = - .975
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20

The Pearson r

$$r = \frac{\Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{N}}{\sqrt{\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]\left[\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right]}}$$

We Need:

- Sum of the Xs ΣX
- Sum of the Ys ΣY
- Sum of the Xs squared $(\Sigma X)^2$
- Sum of the Ys squared $(\Sigma Y)^2$
- Sum of the squared Xs ΣX^2
- Sum of the squared Ys ΣY^2
- Sum of Xs times the Ys ΣXY
- Number of Subjects (N)

Example:

A sample of 6 children was selected, data about their age in years and weight in kilograms was recorded as shown in the following table . Find the correlation between age and weight.

serial No	Age (years)	Weight (Kg)
1	7	12
2	6	8
3	8	12
4	5	10
5	6	11
6	9	13

Serial n.	Age (years) (x)	Weight (Kg) (y)	xy	X^2	Y^2
1	7	12	84	49	144
2	6	8	48	36	64
3	8	12	96	64	144
4	5	10	50	25	100
5	6	11	66	36	121
6	9	13	117	81	169
Total	∑x= 41	∑y= 66	∑xy= 461	∑x2= 291	∑y2= 742

$$r = \frac{461 - \frac{41 \times 66}{6}}{\sqrt{\left[291 - \frac{(41)^2}{6}\right]\left[742 - \frac{(66)^2}{6}\right]}}$$

r = 0.759

strong direct correlation

EXAMPLE: Relationship between Anxiety and Test Scores

Anxiety (X)	Test score (Y)	X^2	Y^2	XY
10	2	100	4	20
8	3	64	9	24
2	9	4	81	18
1	7	1	49	7
5	6	25	36	30
6	5	36	25	30
$\sum X = 32$	$\sum Y = 32$	$\sum X^2 = 230$	$\sum Y^2 = 204$	$\sum XY=129$

Calculating Correlation Coefficient

$$r = \frac{(6)(129) - (32)(32)}{\sqrt{\left(6(230) - 32^2\right)\left(6(204) - 32^2\right)}} = \frac{774 - 1024}{\sqrt{(356)(200)}} = -.94$$

r = - 0.94

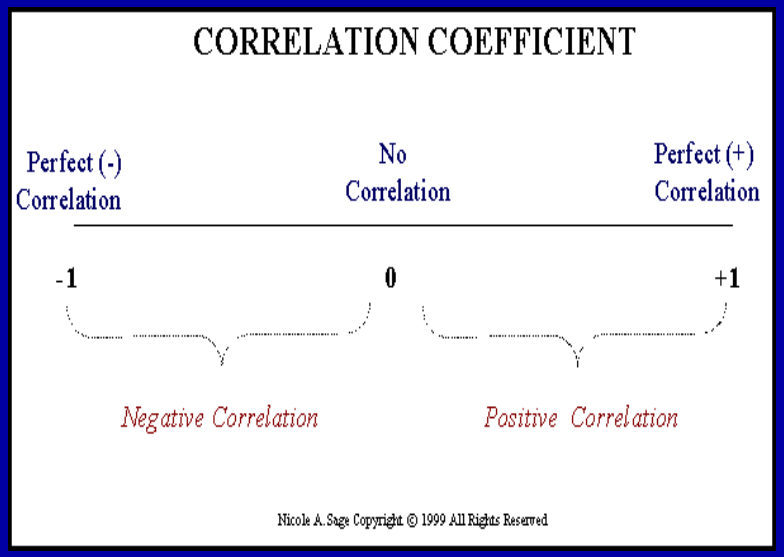
Indirect strong correlation

Correlation Coefficient

a correlation coefficient (r) provides a quantitative way to express the degree of linear relationship between two variables.

- Range: r is always between -1 and 1
- Sign of correlation indicates direction:
  - high with high and low with low -> positive
  - high with low and low with high -> negative
  - no consistent pattern -> near zero

- Magnitude (absolute value) indicates strength (-.9 is just as strong as .9)
  - .10 to .40 weak
  - .40 to .80 moderate
  - .80 to .99 high
  - 1.00 perfect



About “r”

- r is not dependent on the units in the problem
- r ignores the distinction between explanatory and response variables
- r is not designed to measure the strength of relationships that are not approximately straight line
- r can be strongly influenced by outliers

Correlation Coefficient: Limitations

1. Correlation coefficient is appropriate measure of relation only when relationship is linear
2. Correlation coefficient is appropriate measure of relation when equal ranges of scores in the sample and in the population.
3. Correlation doesn't imply causality
   - Using U.S. cities a cases, there is a strong positive correlation between the number of churches and the incidence of violent crime
   - Does this mean churches cause violent crime, or violent crime causes more churches to be built?
   - More likely, both related to population of city (3d variable -- lurking or confounding variable)

Ice-cream sales are strongly correlated with crime rates.

Therefore, ice-cream causes crime.

Without proper interpretation, causation **should not** be assumed, or even implied.

In conclusion !

Z-test will be used for both categorical(qualitative) and quantitative outcome variables.

Student's t-test will be used for only quantitative outcome variables.

Correlation will be used to quantify the linear relationship between two quantitative variables