NORMAL DISTRIBUTION AND ITS APPL ICATION

Objectives of this session:

- Able to understand the concept of Normal distribution.
- Able to calculate the z-score for quantitative variable.
- Able to apply the concepts of normal distribution and z-score in the interpretation of a clinical data.

Problem:

Assume that among diabetics the fasting blood level of glucose is approximately normally distributed with a mean of 105mg per 100ml and an SD of 9 mg per 100 ml. What proportion of diabetics having fasting blood glucose levels between 90 and 125 mg per 100 ml ?

The Normal or Gaussian distribution is the most important continuous probability distribution in statistics.

The term “Gaussian” refers to ‘Carl Freidrich Gauss’ who develop this distribution.

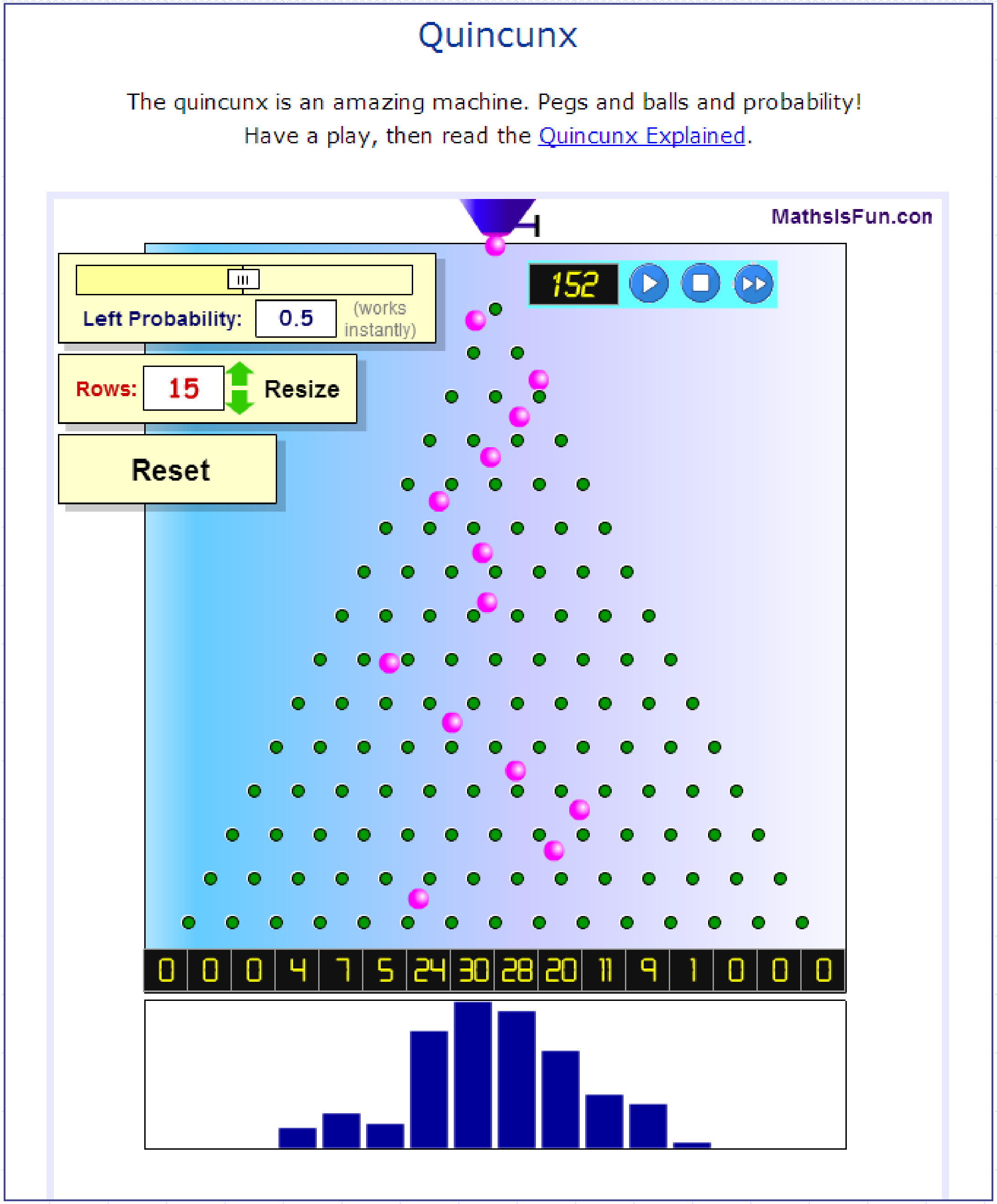
The word ‘normal’ here does not mean ‘ordinary’ or ‘common’ nor does it mean ‘disease-free’.

It simply means that the distribution confirms to a certain formula and shape.

Gaussian Distribution

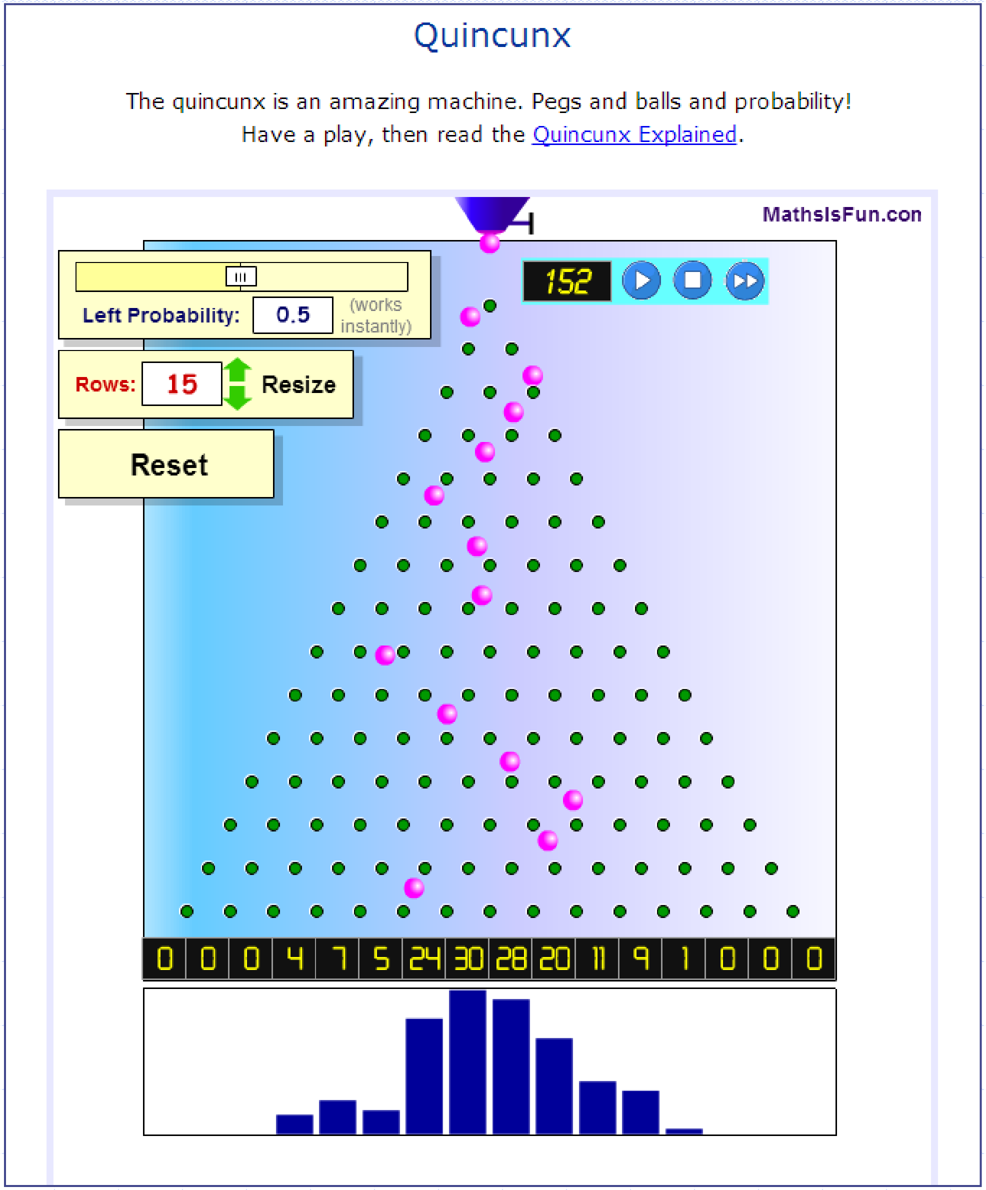
- Many biologic variables follow this pattern
  - **Hemoglobin, Cholesterol, Serum Electrolytes, Blood pressures, age, weight, height**
- One can use this information to define what is normal and what is extreme
- In clinical medicine 95% or 2 Standard deviations around the mean is normal
  - **Clinically, 5% of "normal" individuals are labeled as extreme/abnormal**
    - **We just accept this and move on.**

Pascal Triangle (Quincunx)



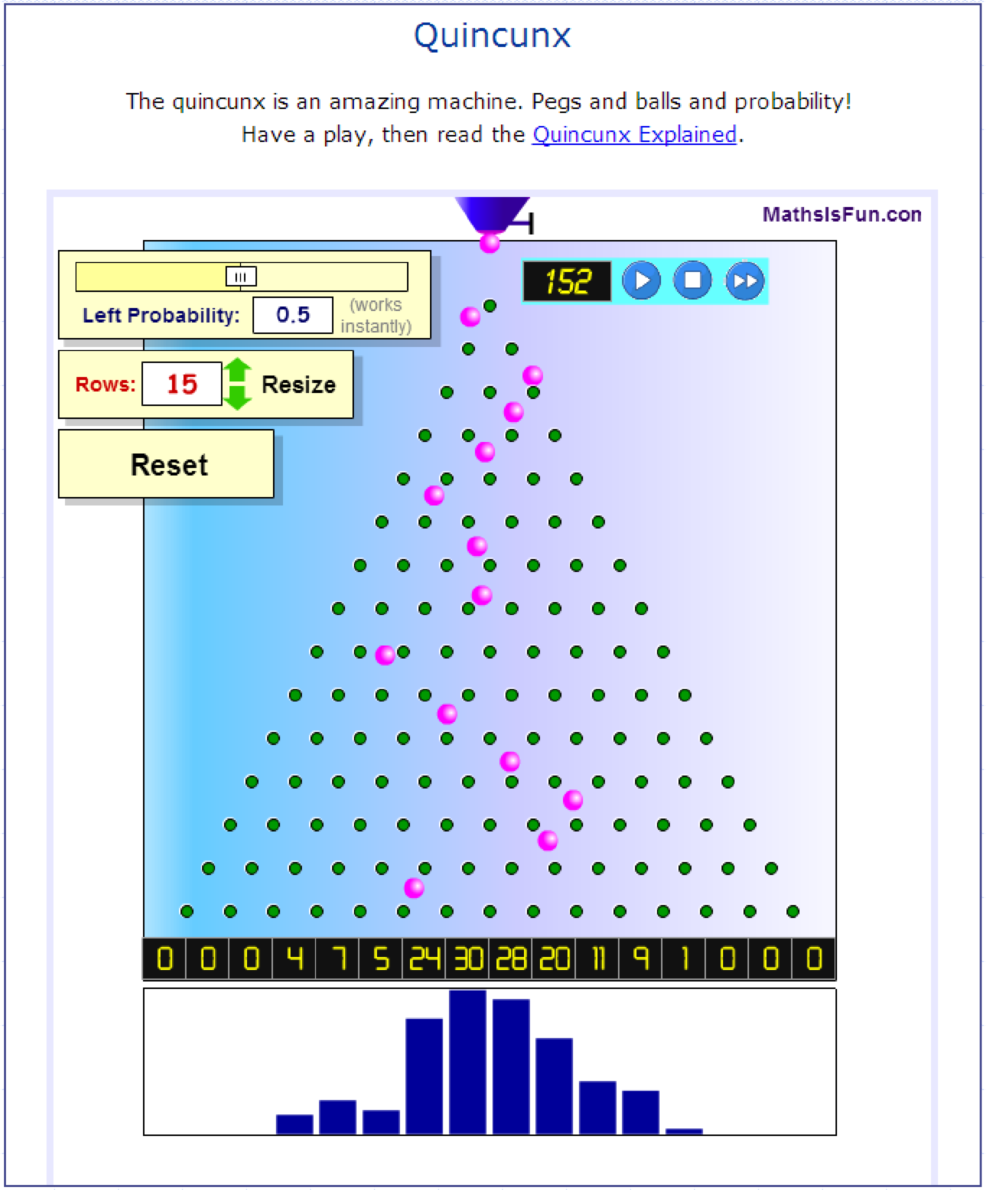
- In the past this experiment was done in a physical box, but today we can use a computer simulation
- Nails were punched into a box to form a triangular shape.

Pascal Triangle (Quincunx)



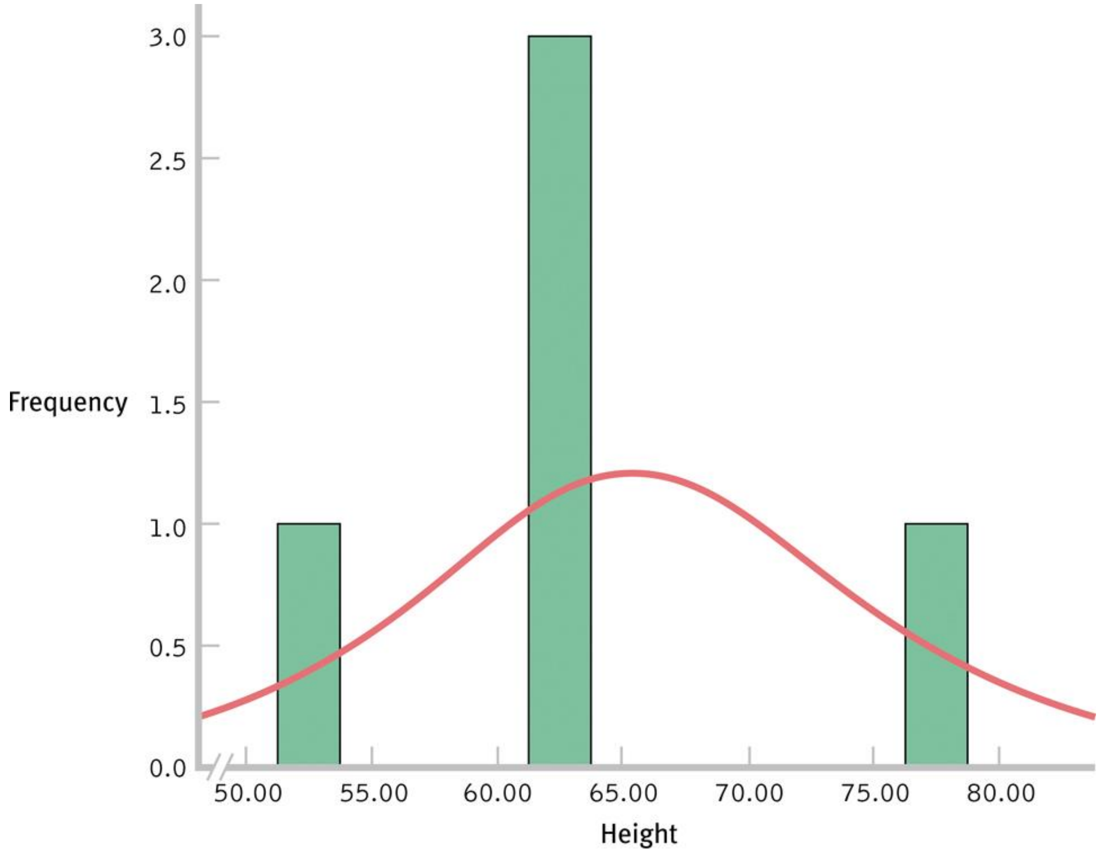
- On top there is only one nail. The second row has two nails. Each subsequent row has one additional nail.
- When a ball is poured into the box from top and lands on the first nail, the probability of going to the left is .5 and to the right is also .5.

Pascal Triangle

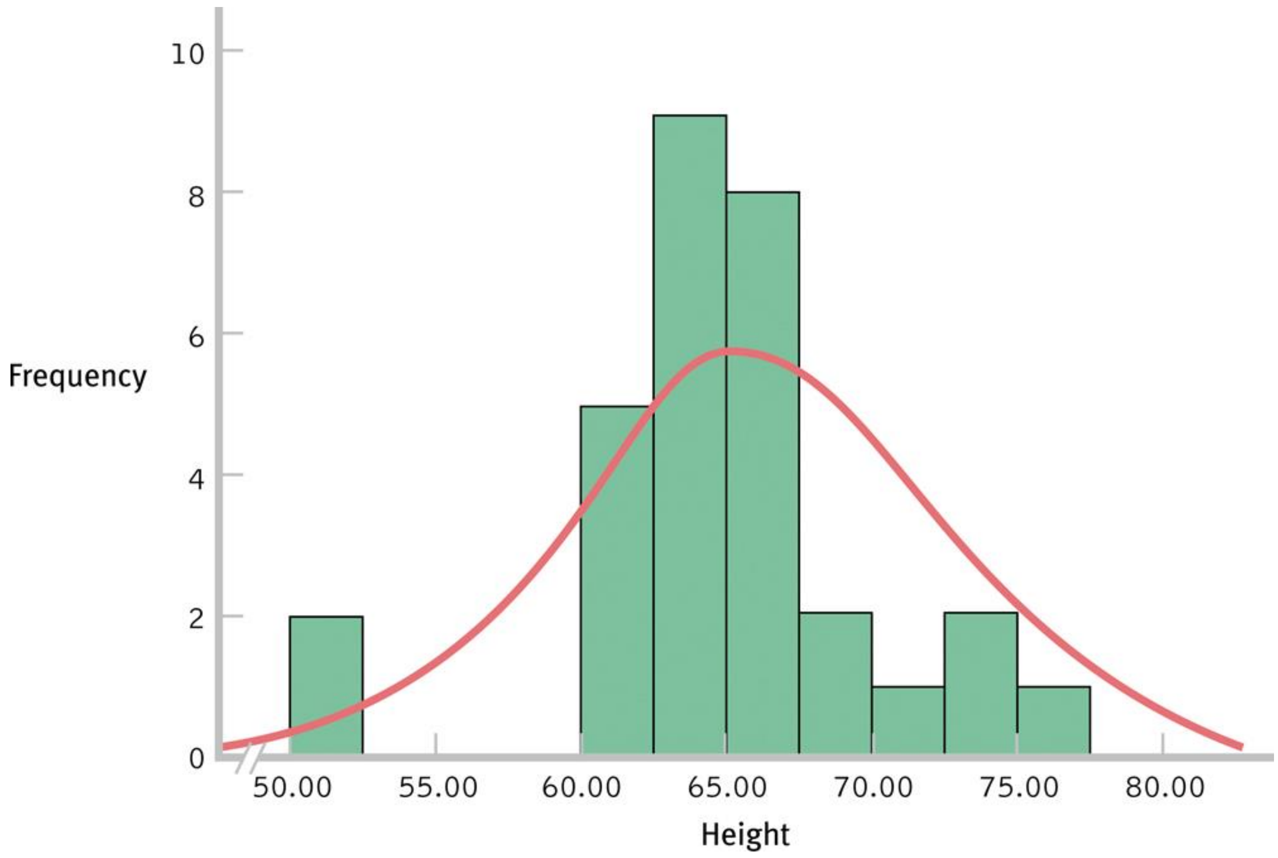


- Subsequently, the probability of going to which direction gets more and more complicated. Nonetheless, the process is random.
- But this random process always produces a normal distribution!
- http://www.mathsisfun.com/data/quincunx.html

Development of a Normal Curve: Sample of 5



Development of a Normal Curve: Sample of 30



Development of a Normal Curve: Sample of 140

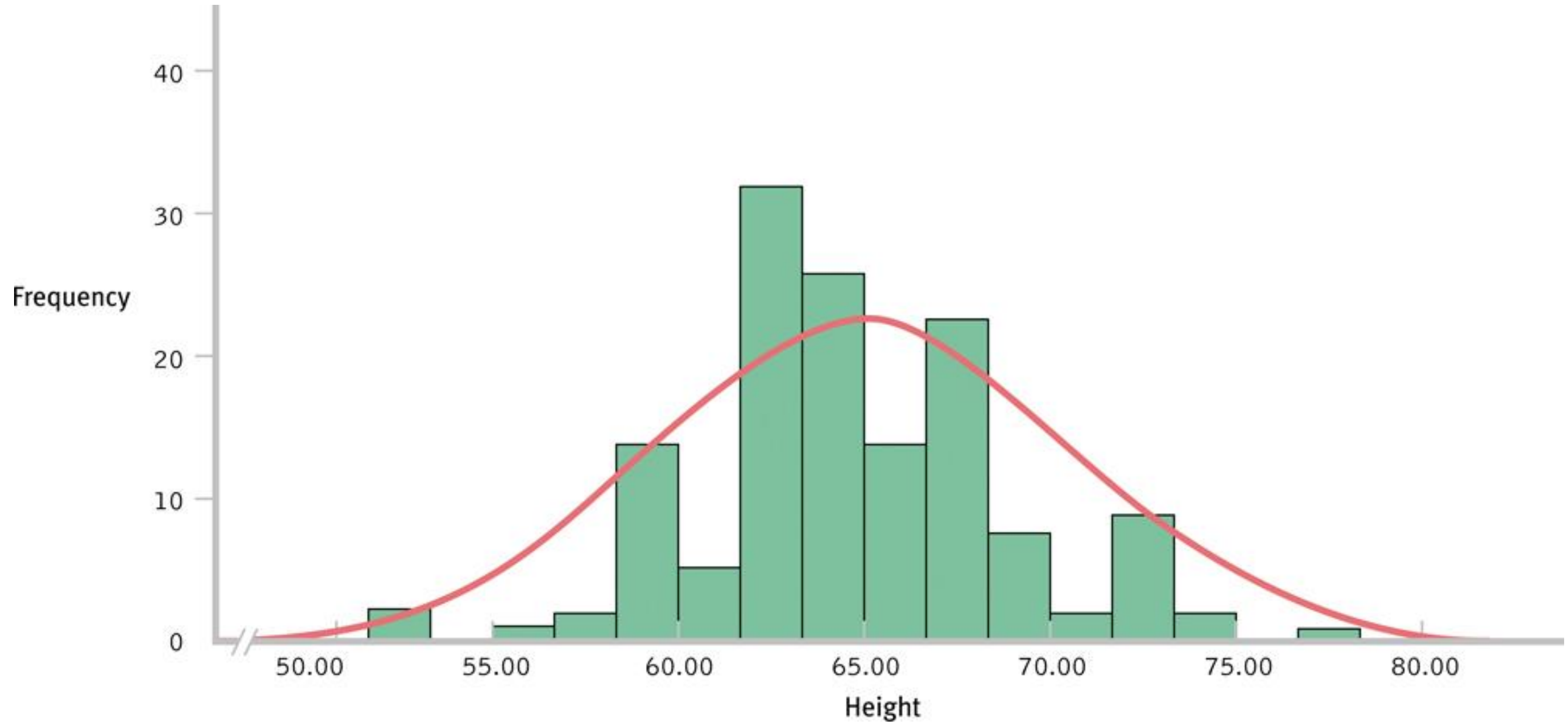


Table 9.3 Example of a Normal Distribution—Distribution of 1000 Men in a Village According to Their Height

Height inches	No. of men of given height			
61–62	2			
62–63	5			
63–64	17			
64–65	43			
65–66	86			
66–67	152			
67–68	193	690	955	997
68–69	197			
69–70	148			
70–71	91			
71–72	45			
72–73	16			
73–74	4			
74–75	1			
Total	1000			

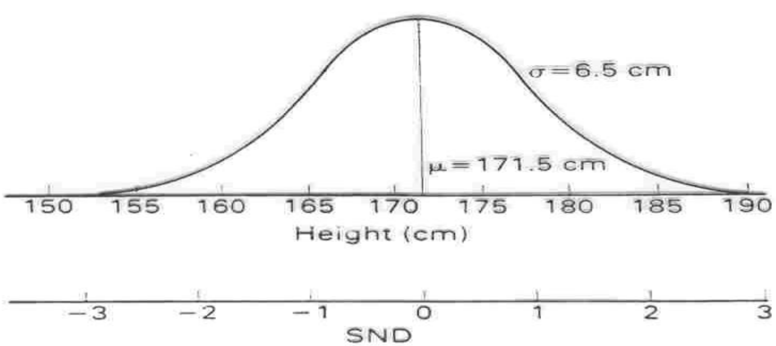
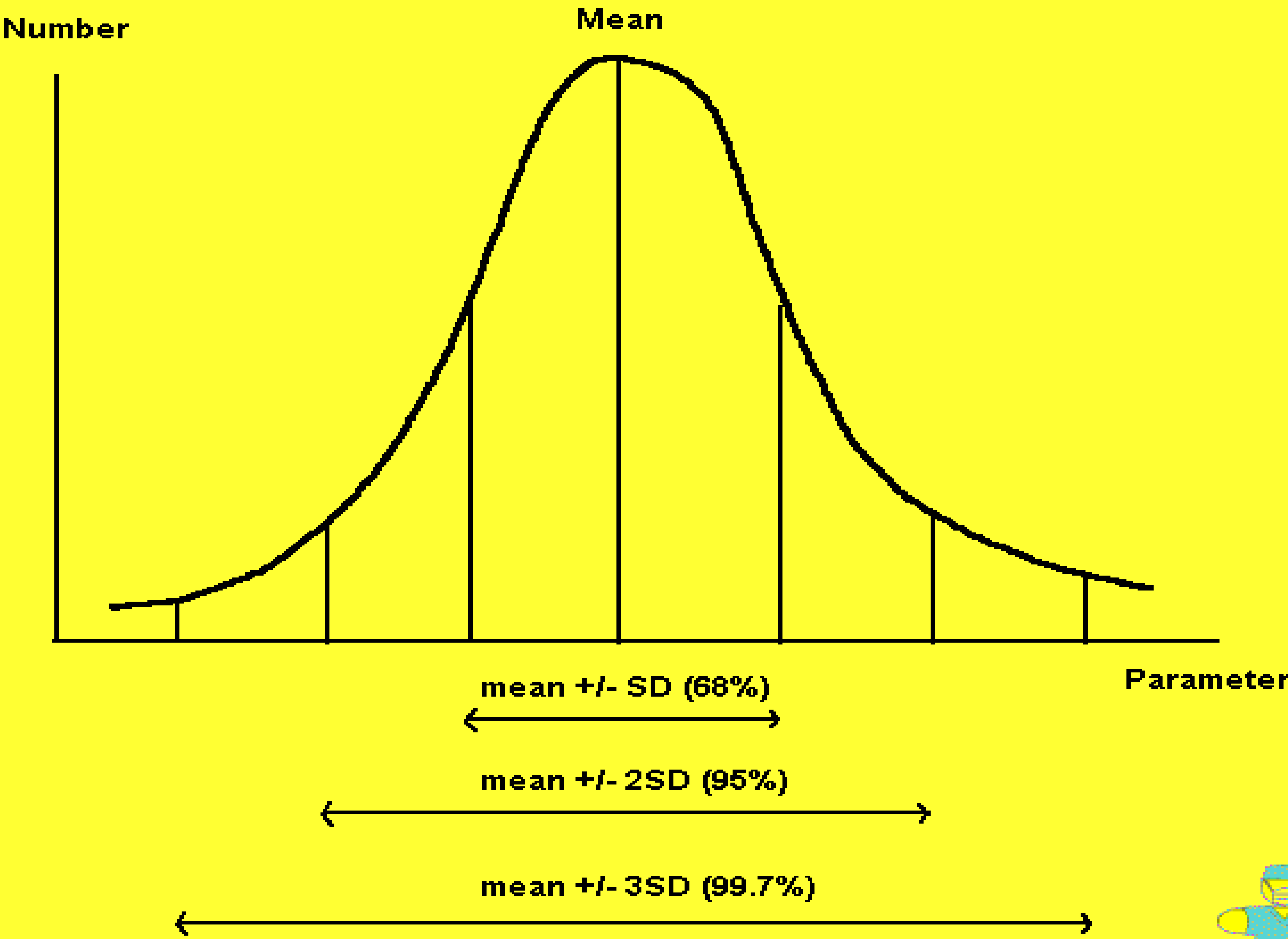
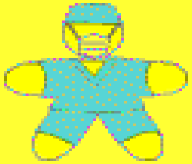


Fig. 5.2 Relationship between normal distribution in original units of measurement and in standard normal deviates. SND = (height − 171.5)/6.5. Height = 171.5 + (6.5 × SND).

Characteristics of Normal Distribution

- Symmetrical about mean, μ
- Mean, median, and mode are equal
- Total area under the curve above the x-axis is one square unit
- 1 standard deviation on both sides of the mean includes approximately 68% of the total area
  - 2 standard deviations includes approximately 95%
  - 3 standard deviations includes approximately 99%


Number
Mean
Parameter
mean +/- SD (68%)
mean +/- 2SD (95%)
mean +/- 3SD (99.7%)



Uses of Normal Distribution

- It’s application goes beyond describing distributions
- It is used by researchers.
- The major use of normal distribution is the role it plays in statistical inference.
- It helps managers to make decisions.

What’s so Great about the Normal Distribution?

- If you know two things,
  - Mean
  - Standard deviation
- you know everything about the distribution
- You know the probability of any value arising

Standardised Scores

- My diastolic blood pressure is 100
  - So what ?
- Normal is 90 (for my age and sex)
  - Mine is high
    - But how much high?
- Express it in standardised scores
  - How many SDs above the mean is that?

- Mean = 90, SD = 4 (my age and sex)

$$\frac{\text{My Score - Mean Score}}{\text{SD}} = \frac{100\text{-}90}{4} = 2.5$$

- This is a *standardised score*, or *z-score*
- Look z tables (or computer)
  - See how often this high (or higher) score occur

z-scores

When a set of data values are normally distributed, we can standardize each score by converting it into a ***z*-score**.

***z*-scores** make it easier to compare data values measured on different scales.

z-scores

A ***z*-score** reflects how many standard deviations above or below the mean a raw score is.

The ***z*-score** is positive if the data value lies above the mean and negative if the data value lies below the mean.

z-score formula

$$z = \frac{x - \mu}{\sigma}$$

Where x represents an element of the data set, the mean is represented by μ and standard deviation by σ.

Standard Scores

- The Z score makes it possible, under some circumstances, to compare scores that originally had different units of measurement.

Comparing Apples and Oranges

- If we can standardize the raw scores on two different scales, converting both scores to z scores, we can then compare the scores

Using z Scores to Make Comparisons

- If you know your score on an exam, and a friend’s score on an exam, you can convert to z scores to determine who did better and by how much.
- z scores are standardized, so they can be compared!

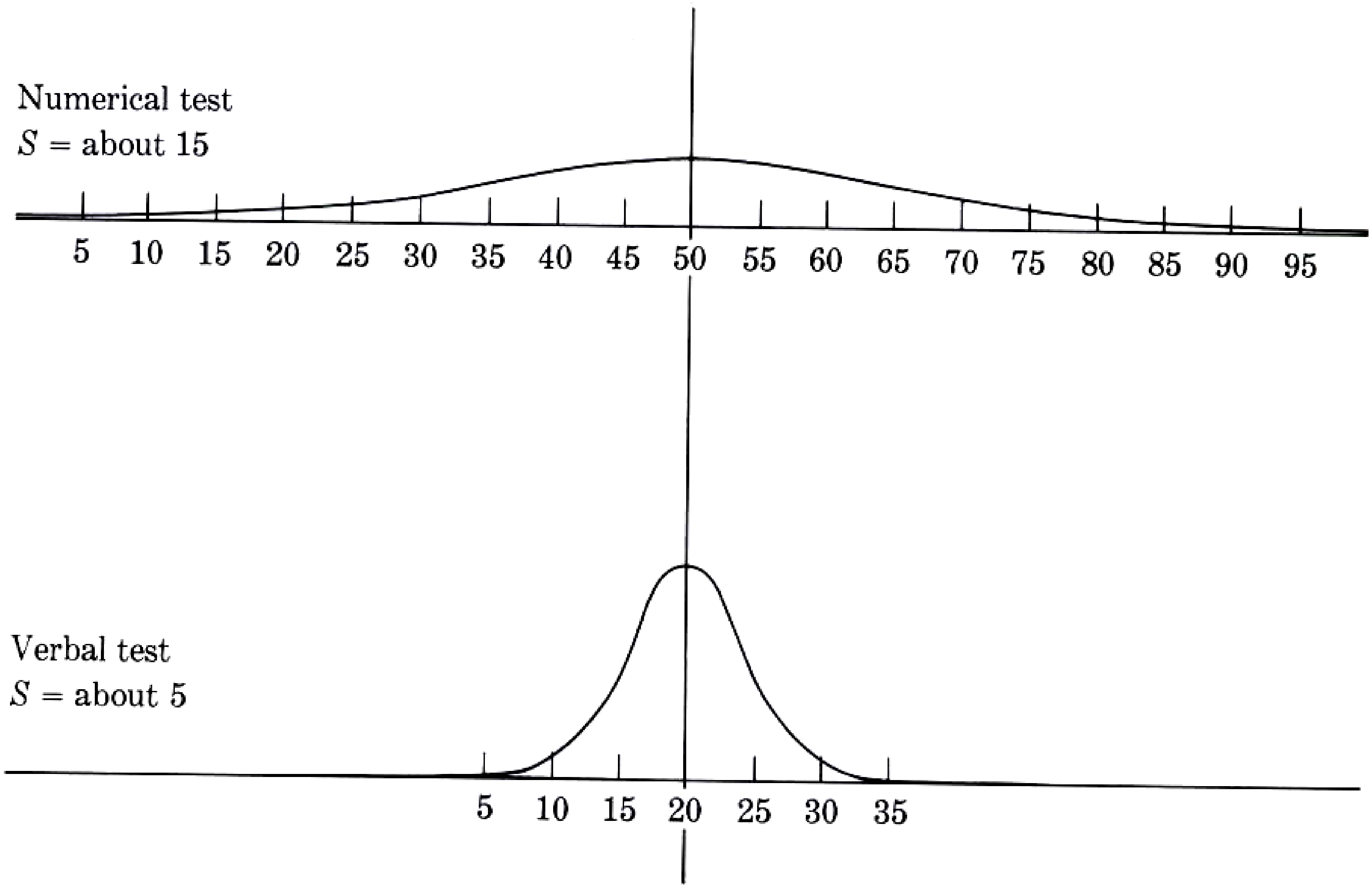
Z Score

- Suppose you scored a 60 on a numerical test and a 30 on a verbal test. On which test did you perform better?
  - First, we need to know how other people did on the same tests.
    - Suppose that the mean score on the numerical test was 50 and the mean score on the verbal test was 20.
    - You scored 10 points above the mean on each test.
    - Can you conclude that you did equally well on both tests?
    - You do not know, because you do not know if 10 points on the numerical test is the same as 10 points on the verbal test.

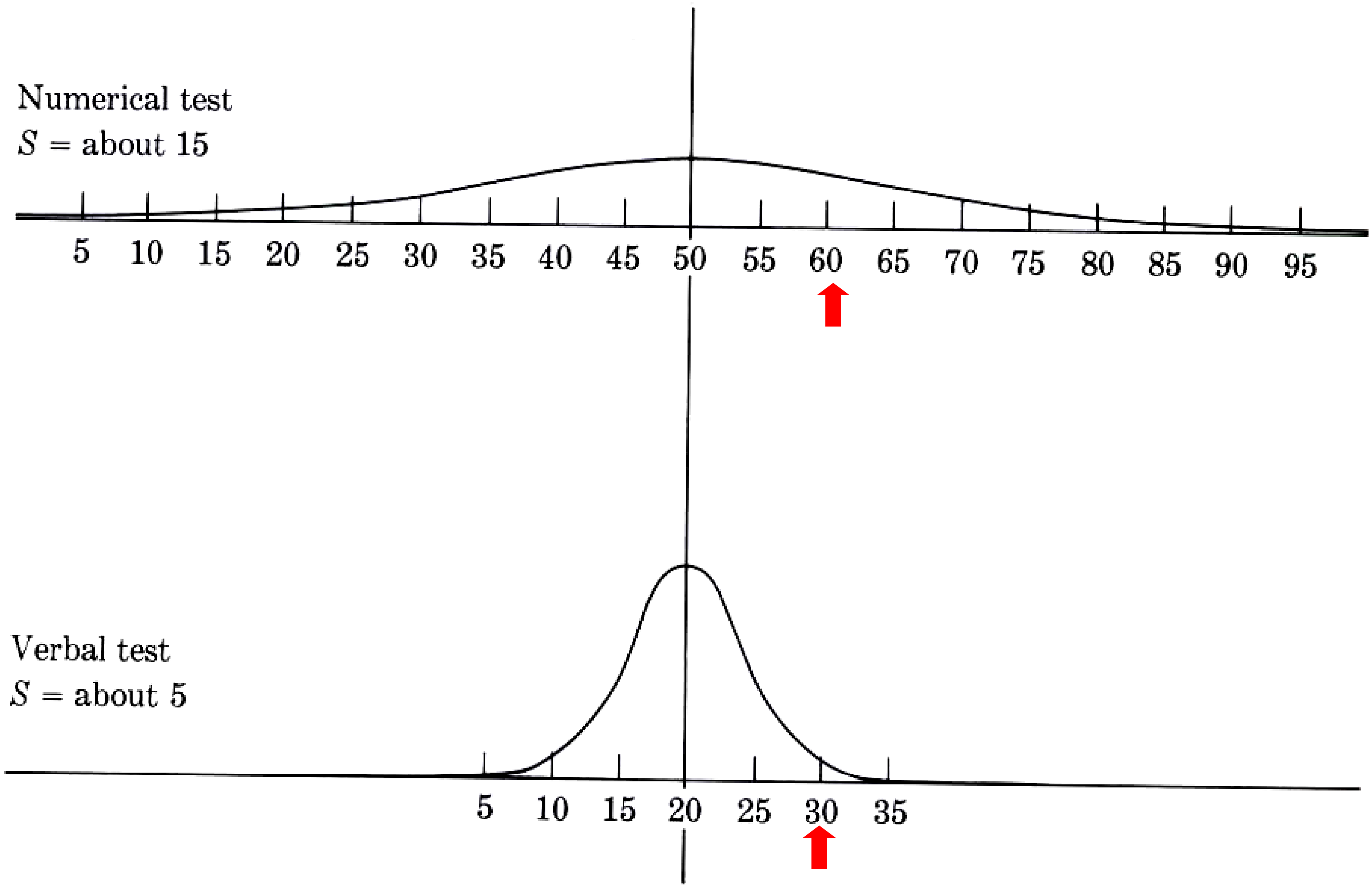
Z Score

- Suppose you scored a 60 on a numerical test and a 30 on a verbal test. On which test did you perform better?
  - Suppose also that the standard deviation on the numerical test was 15 and the standard deviation on the verbal test was 5.
    - Now can you determine on which test you did better?

Z Score


Numerical test
S = about 15
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Verbal test
S = about 5
5 10 15 20 25 30 35

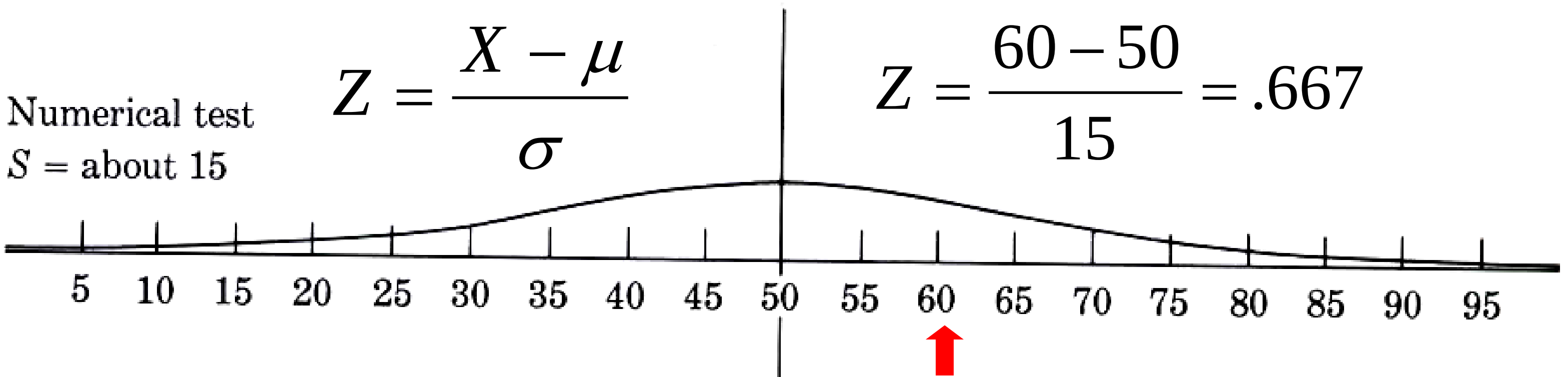
Z Score


Numerical test
S = about 15
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Verbal test
S = about 5
5 10 15 20 25 30 35

Z Score

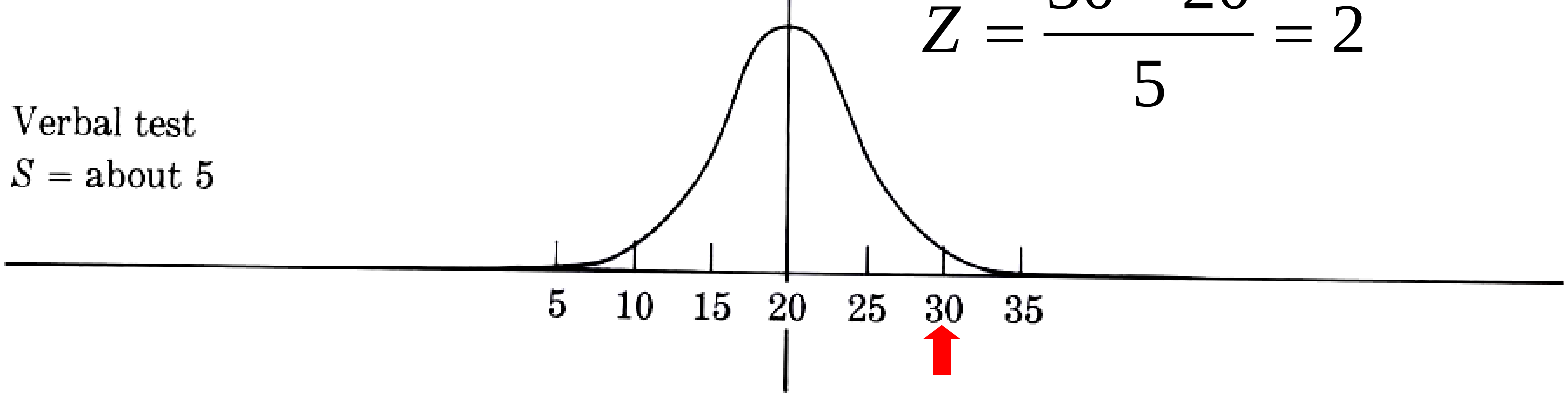
$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{60 - 50}{15} = .667$$



In relation to the rest of the people who took the tests, you did better on the verbal test than the numerical test.

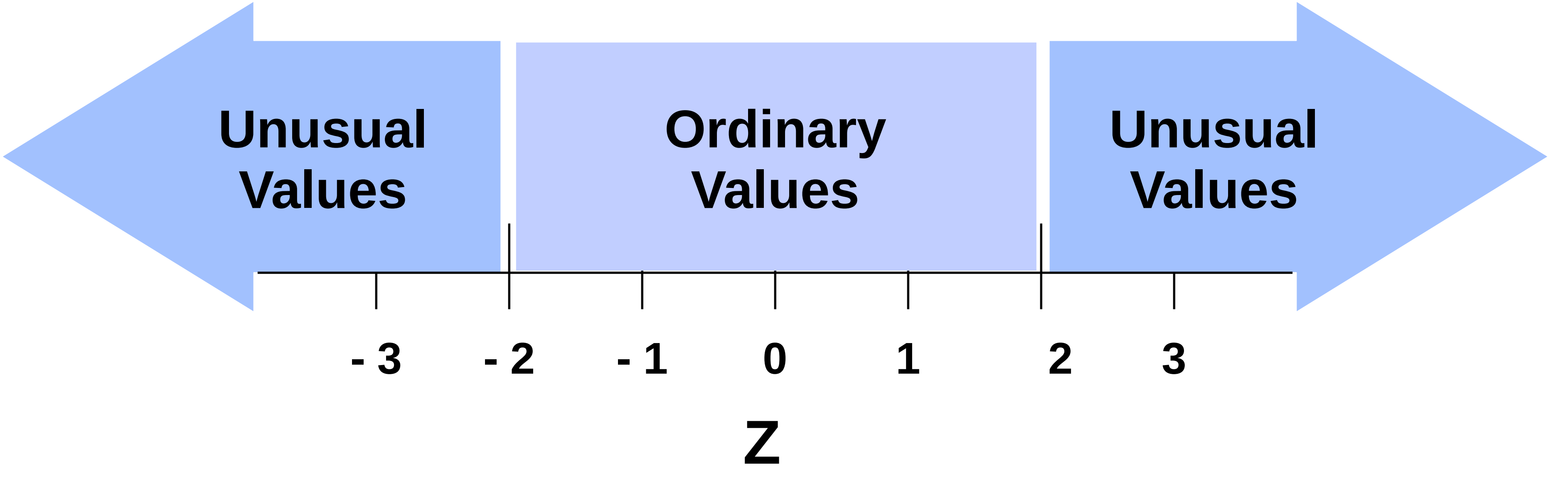
$$Z = \frac{30 - 20}{5} = 2$$



Z score

- Allows you to describe a particular score in terms of where it fits into the overall group of scores.
  - Whether it is above or below the average and how much it is above or below the average.
- A standard score that states the position of a score in relation to the mean of the distribution, using the standard deviation as the unit of measurement.
  - The number of standard deviations a score is above or below a mean.

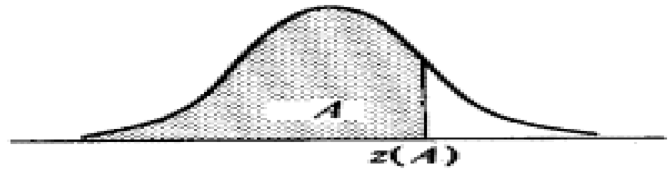
Interpreting Z Scores



The Standard Normal Table

- Using the standard normal table, you can find the area under the curve that corresponds with certain scores.
- The area under the curve is proportional to the frequency of scores.
- The area under the curve gives the probability of that score occurring.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

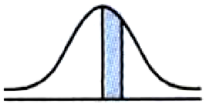
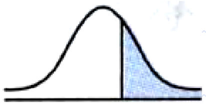
Standard Normal Table

Table A[a] (Continued)
PROPORTIONS OF AREA UNDER STANDARD NORMAL CURVE FOR VALUES OF *z*

A z	B	C	A z	B	C	A z	B	C
1.68	.4535	.0465	2.24	.4875	.0125	2.80	.4974	.0026
1.69	.4545	.0455	2.25	.4878	.0122	2.81	.4975	.0025
1.70	.4554	.0446	2.26	.4881	.0119	2.82	.4976	.0024
1.71	.4564	.0436	2.27	.4884	.0116	2.83	.4977	.0023
1.72	.4573	.0427	2.28	.4887	.0113	2.84	.4977	.0023
1.73	.4582	.0418	2.29	.4890	.0110	2.85	.4978	.0022
1.74	.4591	.0409	2.30	.4893	.0107	2.86	.4979	.0021
1.75	.4599	.0401	2.31	.4896	.0104	2.87	.4979	.0021
1.76	.4608	.0392	2.32	.4898	.0102	2.88	.4980	.0020
1.77	.4616	.0384	2.33	.4901	.0099	2.89	.4981	.0019
1.78	.4625	.0375	2.34	.4904	.0096	2.90	.4981	.0019
1.79	.4633	.0367	2.35	.4906	.0094	2.91	.4982	.0018
1.80	.4641	.0359	2.36	.4909	.0091	2.92	.4982	.0018
1.81	.4649	.0351	2.37	.4911	.0089	2.93	.4983	.0017
1.82	.4656	.0344	2.38	.4913	.0087	2.94	.4984	.0016
1.83	.4664	.0336	2.39	.4916	.0084	2.95	.4984	.0016
1.84	.4671	.0329	2.40	.4918	.0082	2.96	.4985	.0015
1.85	.4678	.0322	2.41	.4920	.0080	2.97	.4985	.0015
1.86	.4686	.0314	2.42	.4922	.0078	2.98	.4986	.0014
1.87	.4693	.0307	2.43	.4925	.0075	2.99	.4986	.0014
1.88	.4699	.0301	2.44	.4727	.0073	3.00	.4987	.0013
1.89	.4706	.0294	2.45	.4929	.0071	3.01	.4987	.0013
1.90	.4713	.0287	2.46	.4931	.0069	3.02	.4987	.0013
1.91	.4719	.0281	2.47	.4932	.0068	3.03	.4988	.0012
1.92	.4726	.0274	2.48	.4934	.0066	3.04	.4988	.0012
1.93	.4732	.0268	2.49	.4936	.0064	3.05	.4989	.0011
1.94	.4738	.0262	2.50	.4938	.0062	3.06	.4989	.0011
1.95	.4744	.0256	2.51	.4940	.0060	3.07	.4989	.0011
1.96	.4750	.0250	2.52	.4941	.0059	3.08	.4990	.0010
1.97	.4756	.0244	2.53	.4943	.0057	3.09	.4990	.0010
1.98	.4761	.0239	2.54	.4945	.0055	3.10	.4990	.0010
1.99	.4767	.0233	2.55	.4946	.0054	3.11	.4991	.0009
2.00	.4772	.0228	2.56	.4948	.0052	3.12	.4991	.0009
2.01	.4778	.0222	2.57	.4949	.0051	3.13	.4991	.0009
2.02	.4783	.0217	2.58	.4951	.0049	3.14	.4992	.0008
2.03	.4788	.0212	2.59	.4952	.0048	3.15	.4992	.0008
2.04	.4793	.0207	2.60	.4953	.0047	3.16	.4992	.0008
2.05	4798	.0202	2.61	.4955	.0045	3.17	.4992	.0008
2.06	.4803	.0197	2.62	.4956	.0044	3.18	.4993	.0007
2.07	.4808	.0192	2.63	.4957	.0043	3.19	.4993	.0007
2.08	.4812	.0188	2.64	.4959	.0041	3.20	.4993	.0007
2.09	.4817	.0183	2.65	.4960	.0040	3.21	.4993	.0007
2.10	.4821	.0179	2.66	.4961	.0039	3.22	.4994	.0006
2.11	.4826	.0174	2.67	.4962	.0038	3.23	.4994	.0006
2.12	.4830	.0170	2.68	.4963	.0037	3.24	.4994	.0006
2.13	.4834	.0166	2.69	.4964	.0036	3.25	.4994	.0006
2.14	.4838	.0162	2.70	.4965	.0035	3.30	.4995	.0005
2.15	.4842	.0158	2.71	.4966	.0034	3.35	.4996	.0004
2.16	.4846	.0154	2.72	.4967	.0033	3.40	.4997	.0003
2.17	.4850	.0150	2.73	.4968	.0032	3.45	.4997	.0003
2.18	.4854	.0146	2.74	.4969	.0031	3.50	.4998	.0002
2.19	.4857	.0143	2.75	.4970	.0030	3.60	.4998	.0002
2.20	.4861	.0139	2.76	.4971	.0029	3.70	.4999	.0001
2.21	.4864	.0136	2.77	.4972	.0028	3.80	.4999	.0001
2.22	.4868	.0132	2.78	.4973	.0027	3.90	.49995	.00005
2.23	.4871	.0129	2.79	.4974	.0026	4.00	.49997	.00003
−z A′	B′	C′	−z A′	B′	C′	−z A′	B′	C′

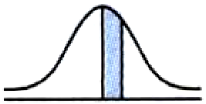
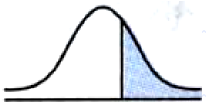
Reading the Z Table

- Finding the proportion of observations between the mean and a score when
  - Z = 1.80

A	B	C
z		
1.68	.4535	.0465
1.69	.4545	.0455
1.70	.4554	.0446
1.71	.4564	.0436
1.72	.4573	.0427
1.73	.4582	.0418
1.74	.4591	.0409
1.75	.4599	.0401
1.76	.4608	.0392
1.77	.4616	.0384
1.78	.4625	.0375
1.79	.4633	.0367
1.80	.4641	.0359
1.81	.4649	.0351
1.82	.4656	.0344
1.83	.4664	.0336
1.84	.4671	.0329
1.85	.4678	.0322
1.86	.4686	.0314
1.87	.4693	.0307
1.88	.4699	.0301
1.89	.4706	.0294
1.90	.4713	.0287
1.91	.4719	.0281
1.92	.4726	.0274

Reading the Z Table

- Finding the proportion of observations above a score when
  - Z = 1.80

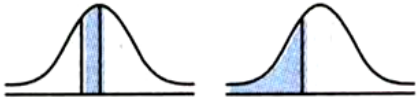
A	B	C
z		
1.68	.4535	.0465
1.69	.4545	.0455
1.70	.4554	.0446
1.71	.4564	.0436
1.72	.4573	.0427
1.73	.4582	.0418
1.74	.4591	.0409
1.75	.4599	.0401
1.76	.4608	.0392
1.77	.4616	.0384
1.78	.4625	.0375
1.79	.4633	.0367
1.80	.4641	.0359
1.81	.4649	.0351
1.82	.4656	.0344
1.83	.4664	.0336
1.84	.4671	.0329
1.85	.4678	.0322
1.86	.4686	.0314
1.87	.4693	.0307
1.88	.4699	.0301
1.89	.4706	.0294
1.90	.4713	.0287
1.91	.4719	.0281
1.92	.4726	.0274

Reading the Z Table

- Finding the proportion of observations between a score and the mean when
  - Z = -2.10

1.98	.4761	.0239
1.99	.4767	.0233
2.00	.4772	.0228
2.01	.4778	.0222
2.02	.4783	.0217
2.03	.4788	.0212
2.04	.4793	.0207
2.05	4798	.0202
2.06	.4803	.0197
2.07	.4808	.0192
2.08	.4812	.0188
2.09	.4817	.0183
2.10	.4821	.0179
2.11	.4826	.0174
2.12	.4830	.0170
2.13	.4834	.0166
2.14	.4838	.0162
2.15	.4842	.0158
2.16	.4846	.0154
2.17	.4850	.0150
2.18	.4854	.0146
2.19	.4857	.0143
2.20	.4861	.0139
2.21	.4864	.0136
2.22	.4868	.0132
2.23	.4871	.0129

−z



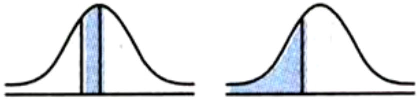
A′ B′ C′

Reading the Z Table

- Finding the proportion of observations below a score when
  - Z = -2.10

1.98	.4761	.0239
1.99	.4767	.0233
2.00	.4772	.0228
2.01	.4778	.0222
2.02	.4783	.0217
2.03	.4788	.0212
2.04	.4793	.0207
2.05	4798	.0202
2.06	.4803	.0197
2.07	.4808	.0192
2.08	.4812	.0188
2.09	.4817	.0183
2.10	.4821	.0179
2.11	.4826	.0174
2.12	.4830	.0170
2.13	.4834	.0166
2.14	.4838	.0162
2.15	.4842	.0158
2.16	.4846	.0154
2.17	.4850	.0150
2.18	.4854	.0146
2.19	.4857	.0143
2.20	.4861	.0139
2.21	.4864	.0136
2.22	.4868	.0132
2.23	.4871	.0129

−z



A′ B′ C′

Z scores and the Normal Distribution

- Can answer a wide variety of questions about any normal distribution with a known mean and standard deviation.
- Will address how to solve two main types of normal curve problems:
  - Finding a proportion given a score.
  - Finding a score given a proportion.

Exercises

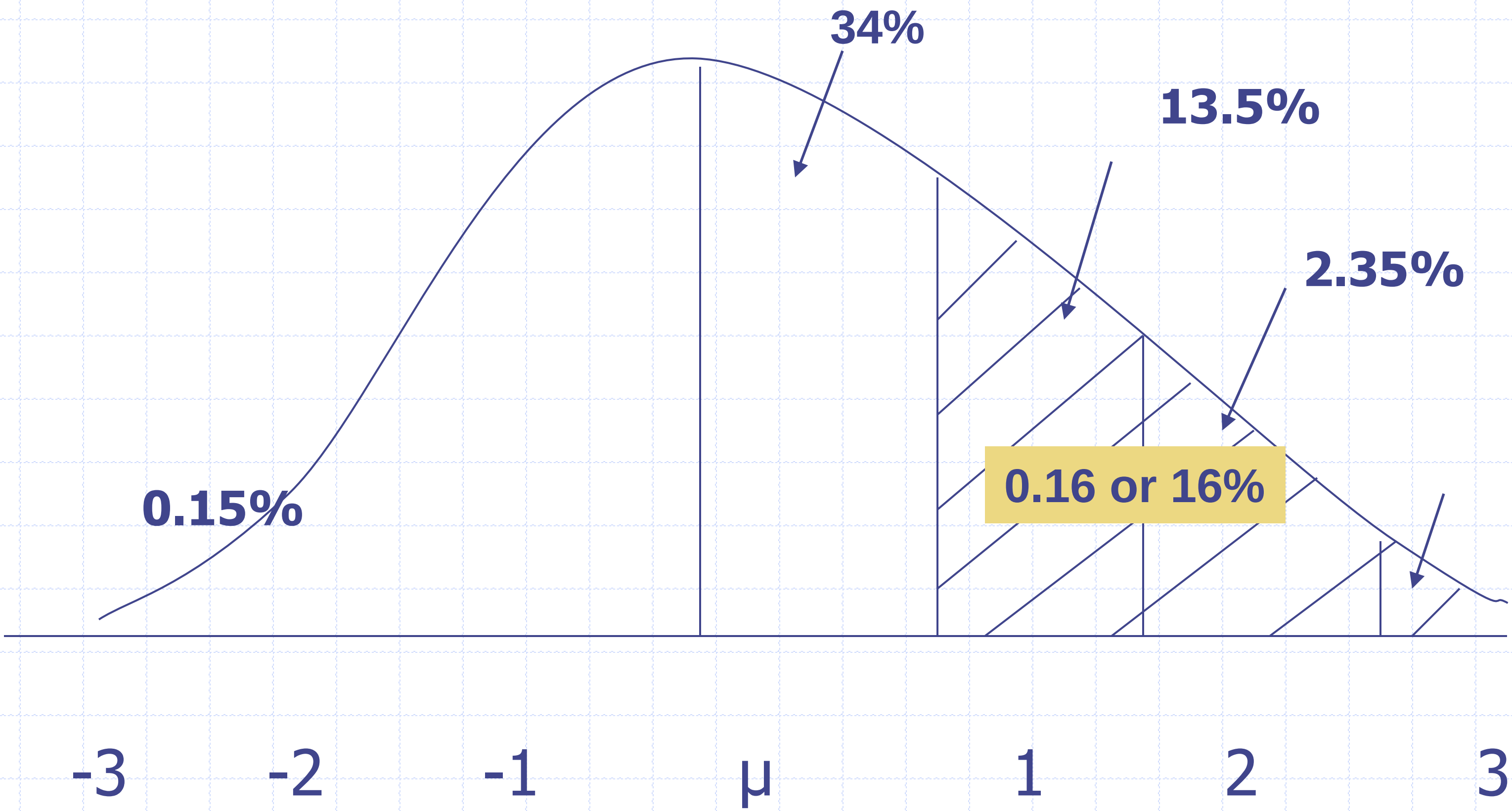
- Assuming the normal heart rate (H.R) in normal healthy individuals is normally distributed with Mean = 70 and Standard Deviation =10 beats/min

Exercise # 1

Then:

1) What area under the curve is above 80 beats/min?

Diagram of Exercise # 1

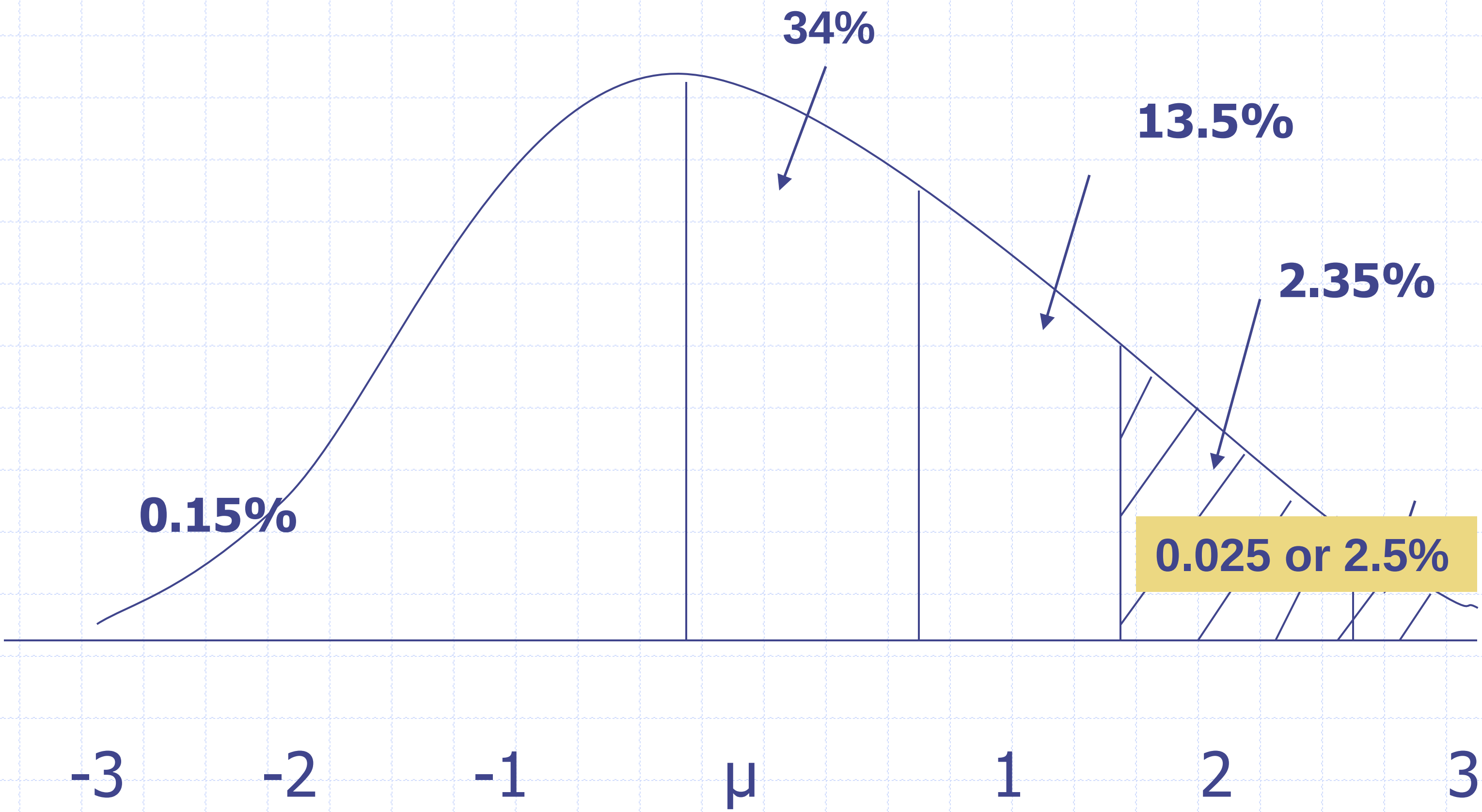


Exercise # 2

Then:

2) What area of the curve is above 90 beats/min?

Diagram of Exercise # 2

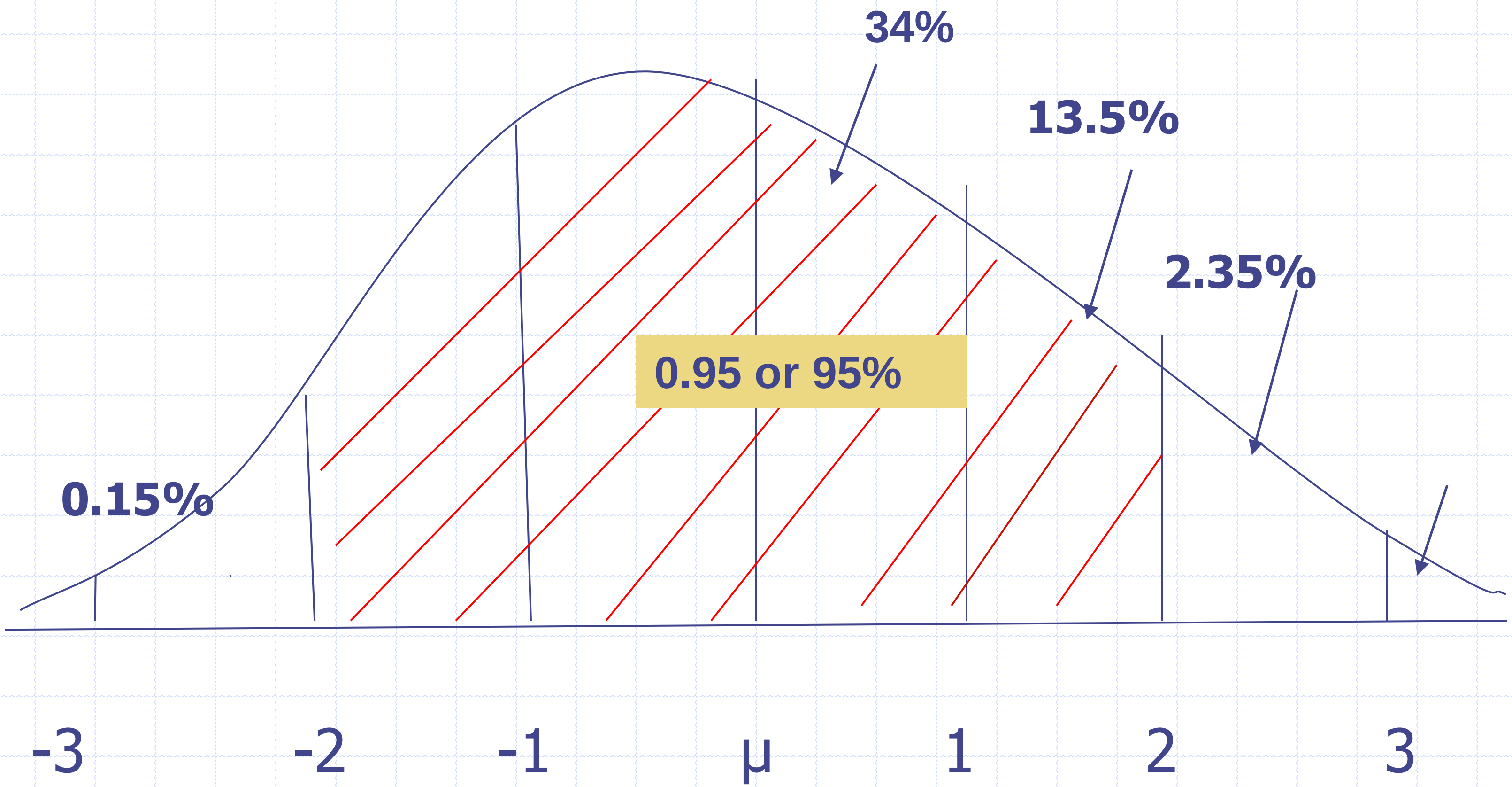


Exercise # 3

Then:

3) What area of the curve is between 50-90 beats/min?

Diagram of Exercise # 3

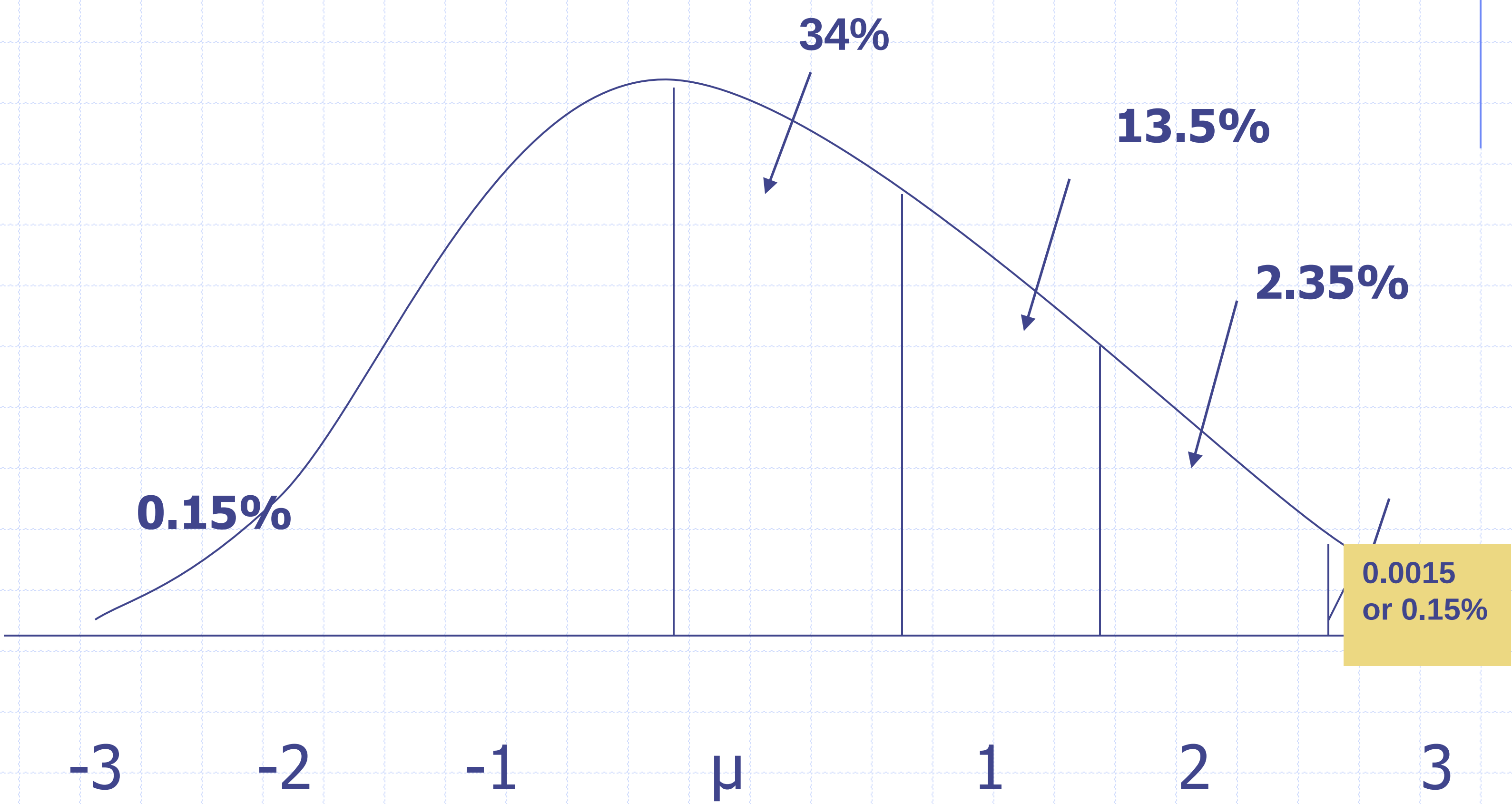


Exercise # 4

Then:

4) What area of the curve is above 100 beats/min?

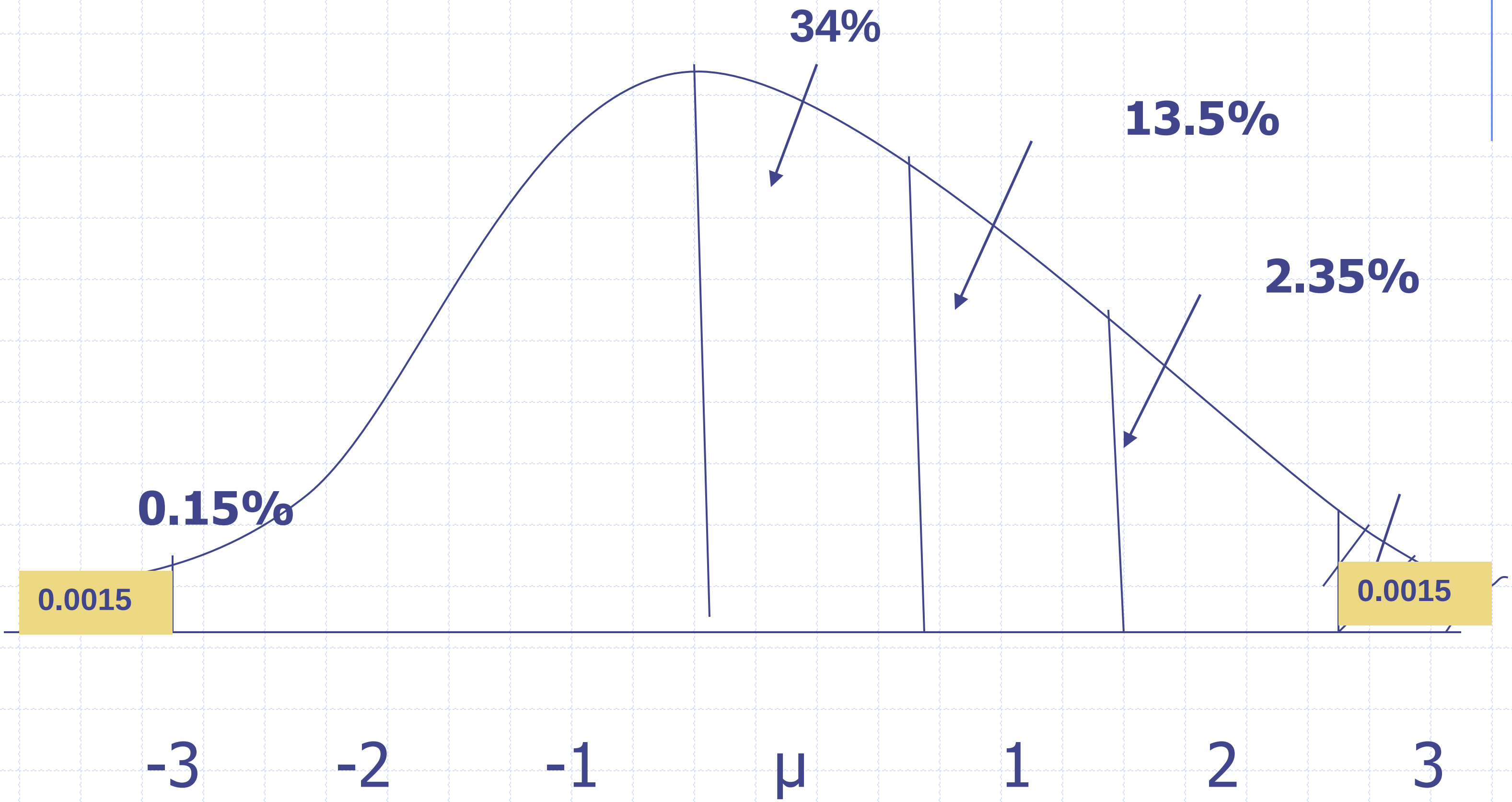
Diagram of Exercise # 4



Exercise # 5

5) What area of the curve is below 40 beats per min or above 100 beats per min?

Diagram of Exercise # 5



Exercise:

- Assuming the normal heart rate (H.R) in normal healthy individuals is normally distributed with Mean = 70 and Standard Deviation =10 beats/min

Then:

1) What area under the curve is above 80 beats/min?

   Ans: 0.16 (16%)

2) What area of the curve is above 90 beats/min?

   Ans: 0.025 (2.5%)

3) What area of the curve is between 50-90 beats/min?

   Ans: 0.95 (95%)

4) What area of the curve is above 100 beats/min?

   Ans: 0.0015 (0.15%)

5) What area of the curve is below 40 beats per min or above 100 beats per min?

   Ans: 0.0015 for each tail or 0.3%

Problem:

Assume that among diabetics the fasting blood level of glucose is approximately normally distributed with a mean of 105mg per 100ml and an SD of 9 mg per 100 ml. What proportion of diabetics having fasting blood glucose levels between 90 and 125 mg per 100 ml ?

iii) What levels encompass the middle 95 per cent of diabetics?

Answers Example 2
Let X be the random variable denoting the fasting blood glucose level. X has a normal distribution with mean = 105 and standard deviation = 9.

i) We have to compute $P(90 \leq X \leq 125)$. The table is available only for the probabilities of a standard normal distribution. Thus we have to convert X to a standard normal variable (Z), using the formula on page 5 of this module.

We require $P(90 \leq X \leq 125)$.

This can be written as

$$P\left[\frac{90-105}{9} \leq \frac{X-105}{9} \leq \frac{125-105}{9}\right] = P(-1.67 \leq Z \leq 2.22)$$

$$\text{since} \quad Z = \frac{X-105}{9}$$

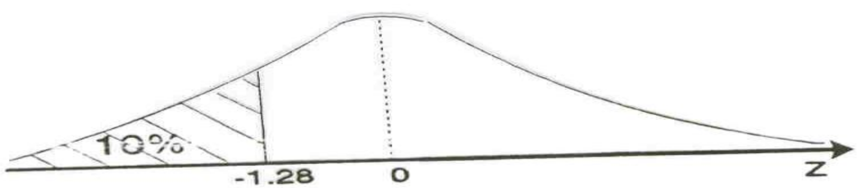
$$= P(Z \leq 2.22) - P(Z < -1.67)$$

$$= 0.9868 - 0.0475$$

$$= 0.9393$$

Therefore 94% of diabetics have fasting blood glucose levels between 90 and 125.

ii)



From the table we know that -1.28 cuts off the lower 10 per cent of the standard normal curve. Now we have to find the corresponding X-value.

ANY

QUESTIONS